The Gathering Problem for Two Oblivious Robots with Unreliable Compasses

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Joint work with
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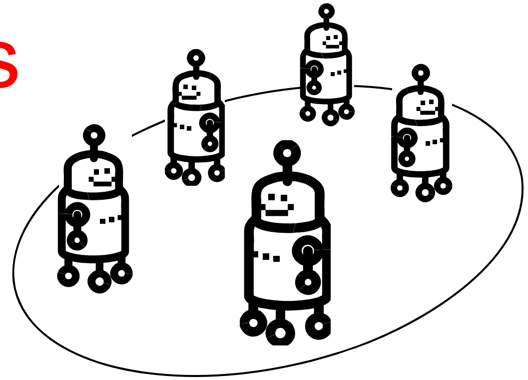
Coordination of Autonomous Mobile Robots

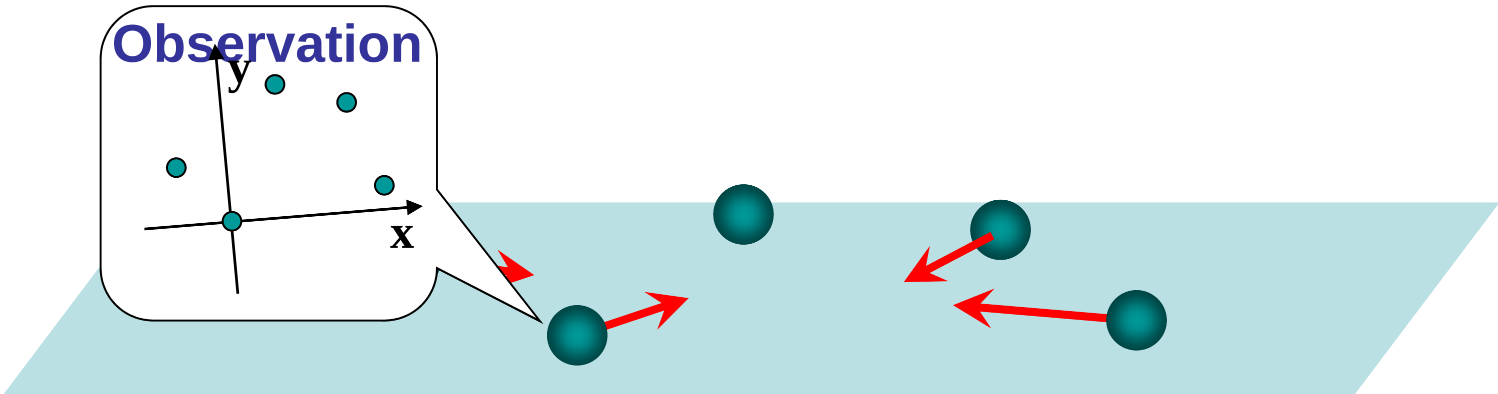
- Autonomous Mobile Robots
  Multiple, Fully decentralized



- Coordination task of Mobile Robots
  - Gathering, Convergence, Formation ...
- Challenges from the theoretical aspect
  - Clarifying the "weakest capability" to solve a given task

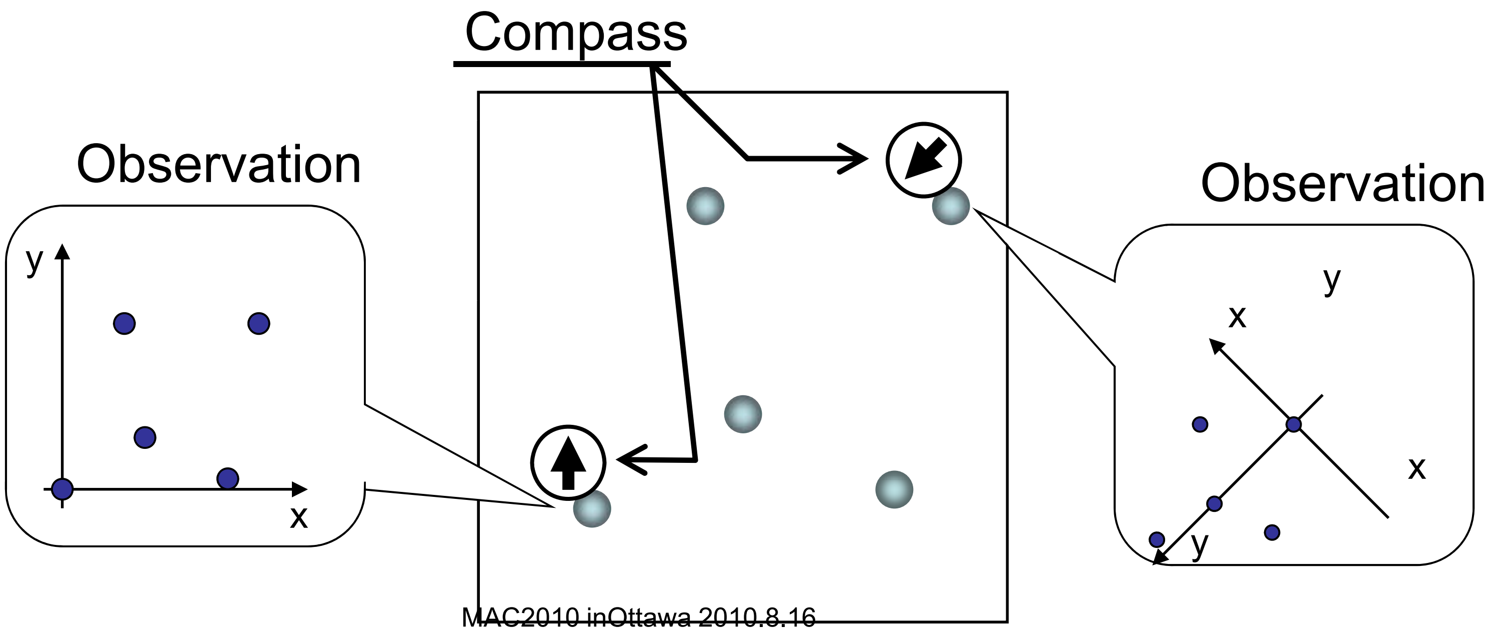
Autonomous Mobile Robots

- Robot: Point on an infinite 2D-space
  - Anonymous (No distinguished ID)
  - Oblivious(No memory)
  - Deterministic
  - No communication (Observe the environment and Move)



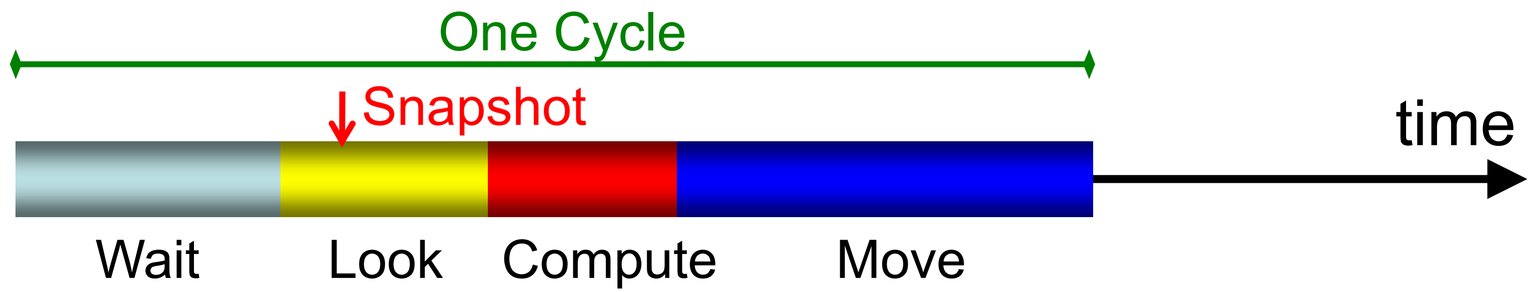
Observation

- Each robot has a local x-y coordinate system(LCS)
  - The current position is the origin
  - The +direction of y-axis follows the local compass
- Agreement level of LCSs depends on the model (compass model)



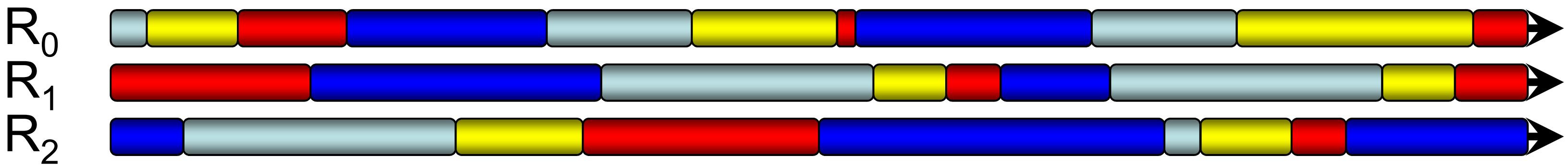
Execution of Robots (Behavior of Each Robot)

- Wait-look-compute-move cycle
  - **Wait**: Idle state
  - **Look**: Take a snapshot of all robots' current locations (in terms of LCS)
  - **Compute**: Deciding the next position
  - **Move**: Move to the next position(unpredictable move)

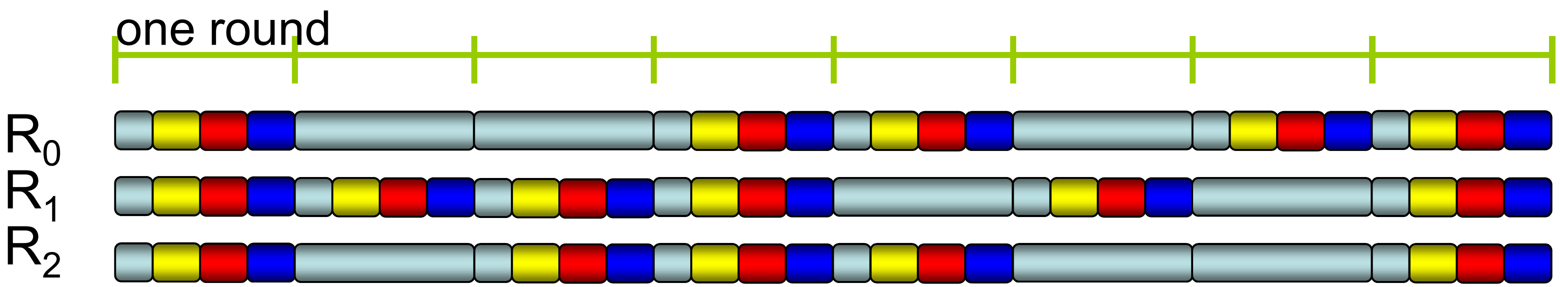


Timing Model(How Cycles are Synchronized)

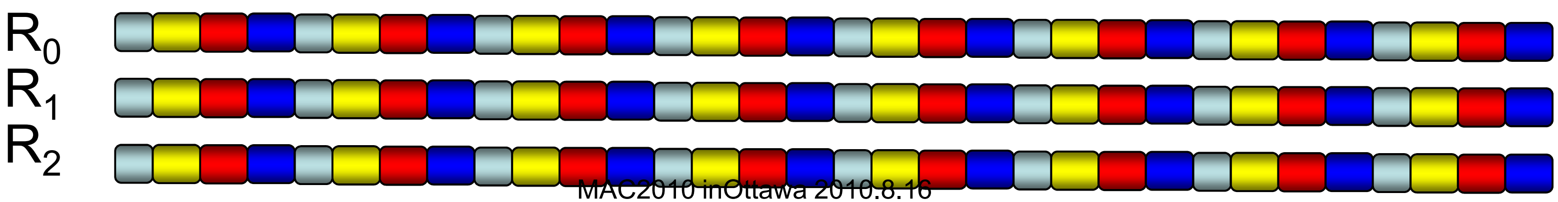
- Asynchronous(CORDA): No bound for length of each step



- Semi-synchronous(SYm, ATOM): Synchronized Round (one cycle=one round)
  - Only a subset of all robots becomes active in each round



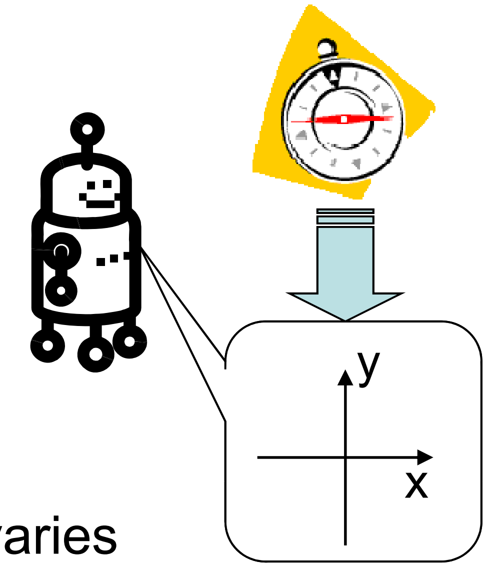
- Synchronous: All robots are completely synchronized



Compass Models

Inaccurate Compasses

- Every robot has its own local coordinate system
  - Compass gives y-axis' positive direction of the local coordinate system.
    - a compass varies, a local coordinate system varies



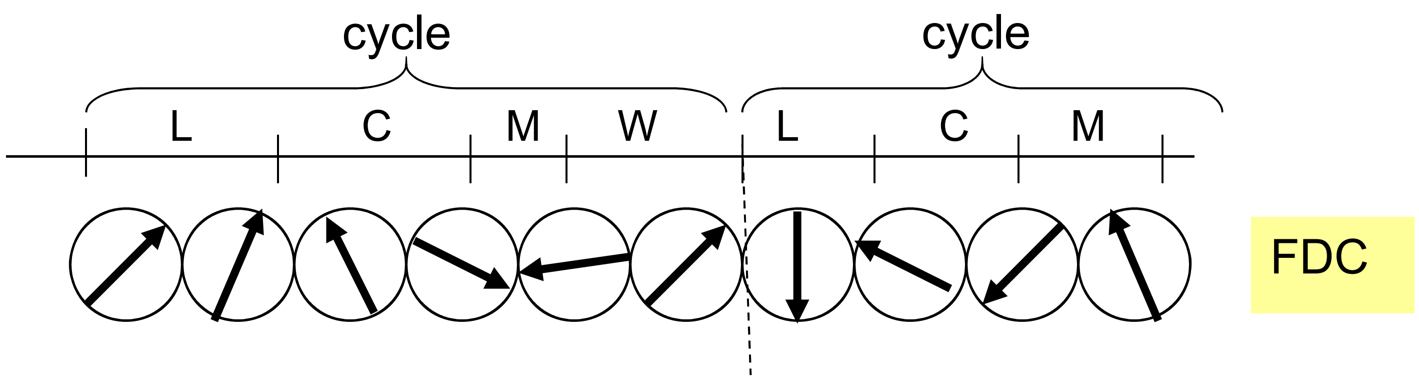
- Inaccuracy of Compass
  - **Variance of Compasses**
    - the variance of indicated directions of compasses
  - **Deviation from the absolute direction**
    - the difference of indicated direction between compasses

Compass Models – Variance –1

- Fully-Dynamic Compass(FDC)
- Semi-Dynamic Compass(SDC)
- FiXed Compass(FXC)

Fully-dynamic Compass (FDC)

A compass whose indicated direction may **vary at any time during execution**.



Gathering is impossible on FDC.

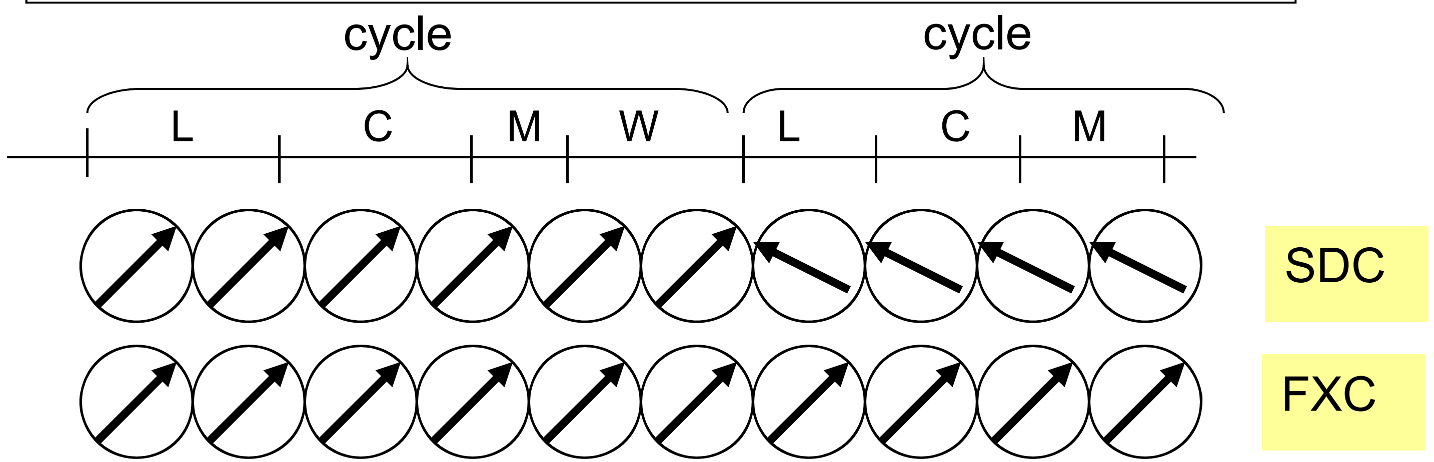
Compass Models – Variance –2

Semi-dynamic Compass (SDC) | **Dynamic Compass**

A compass whose indicated direction may **vary at the time between any two cycles** (never change during one cycle).

Fixed Compass (FXC) | **Static Compass**

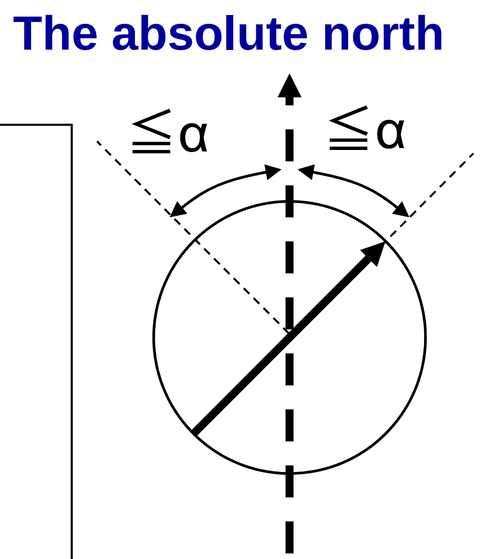
A compass whose indicated direction **never varies**.



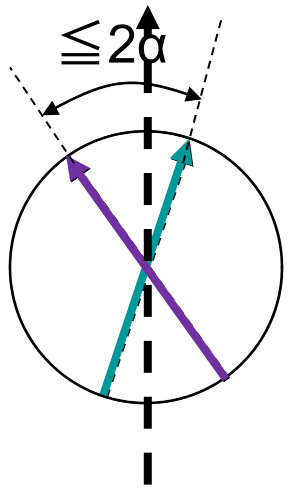
Compass Models –Deviation–

α-error Compass

A direction of “**the absolute north**” is assumed. The each angle which is formed by the indicated direction of robots’ compass and the absolute north is at most α.

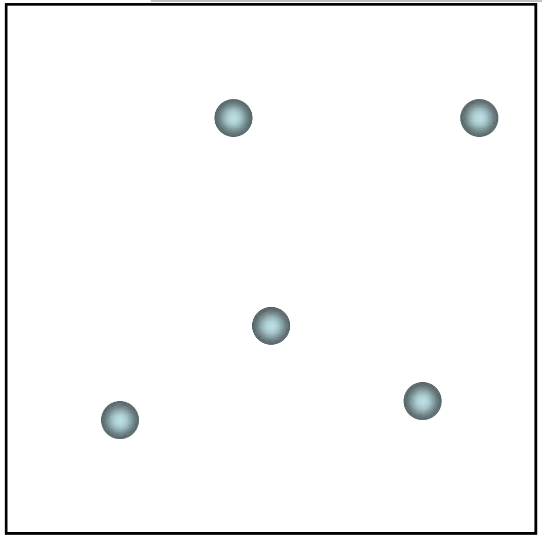


Note that the angle between two robots’ compasses is at most 2α on α-error compass model



Gathering Problem

- All robots meet at one point on a plane
  - Not convergence



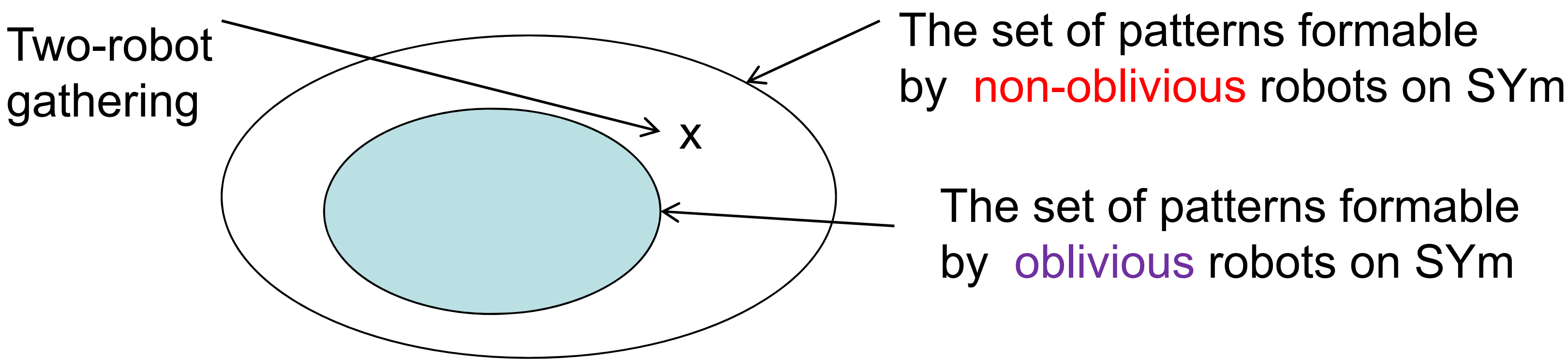
- Known Results
  - Agreed Compass : Solvable
    - CORDA / Arbitrary #robots
  - Disagreed Compass : Unsolvable
    - SYm / #robots = 2

Our results (summary)

Two-robot Gathering problem on α-error compass

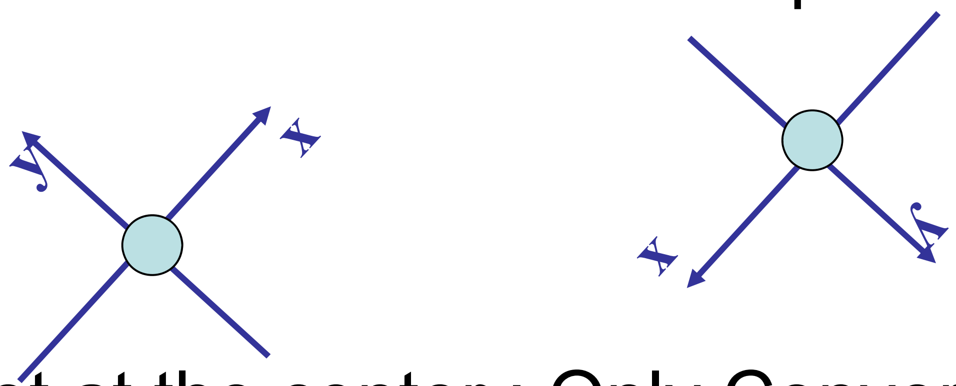
	SYm	CORDA
Semi-DC	impossible(α=π/4) possible(α<π/4)	open possible(α<π/6)
FiXedC	impossible(α=π/2)[1] →	← possible(α<π/2)

[1] I. Suzuki, M. Yamashita, SIAM J. Computing, 28, 4, 1347-1363, 1999.



Impossibility(π/2-error compass , FXC and SYm)

- Opposite directions of two compasses
  - Approach to another : Swap occurs



  - Meet at the center : Only Convergence

Impossibility(π/4-error compass , SDC and SYm)

A necessary condition for any gathering algorithm :

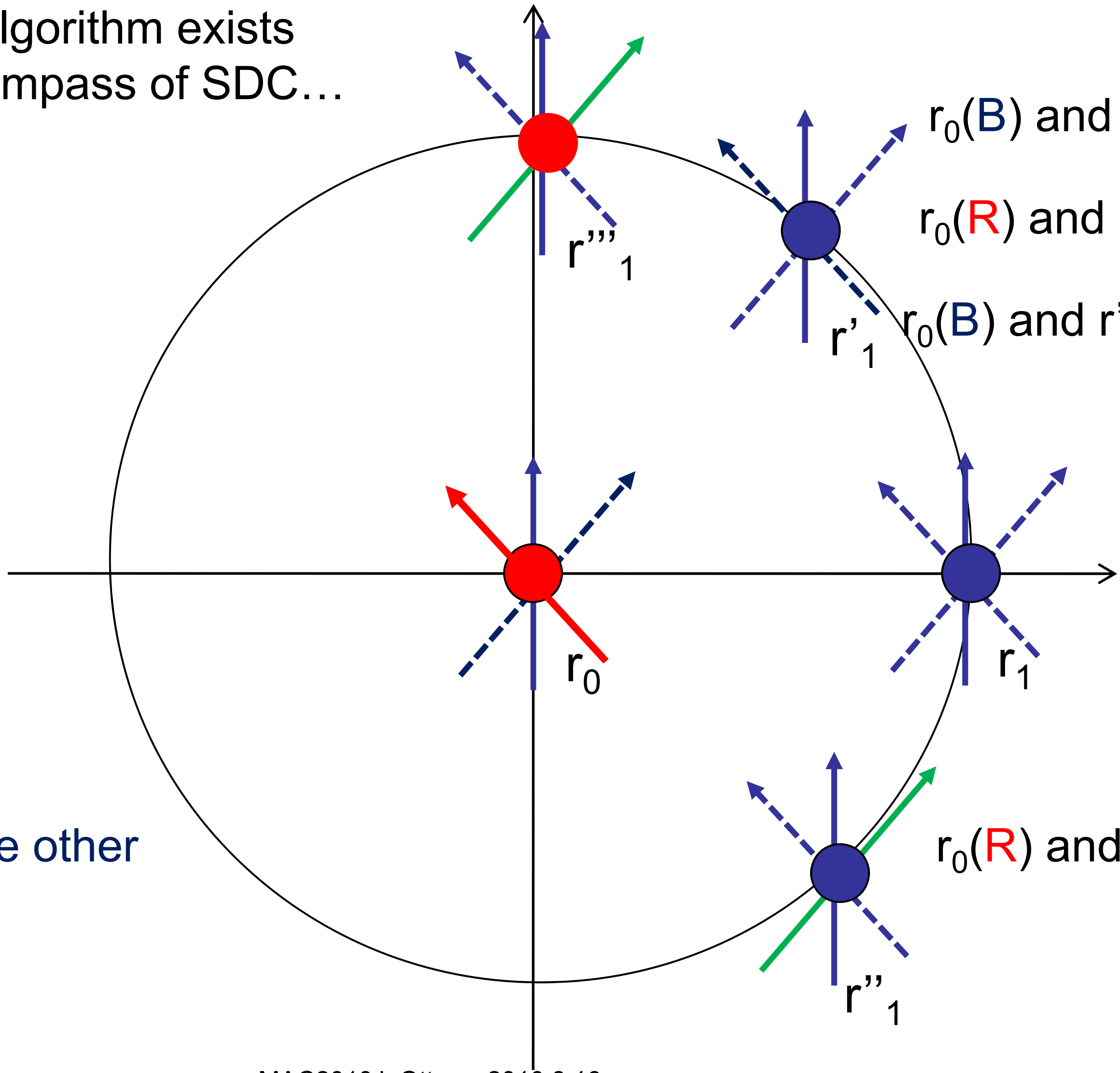
stable configuration

a) There exists a configuration such that
   1) One robot r_0 stays at own position
   2) Another robot r_1 moves to the robot r_0
b) This configuration is regardless of the current local coordinate systems of both robots

Stable configuration

Impossibility(π/4-error compass , SDC and Sym)

If a gathering algorithm exists on π/4-error compass of SDC…



r_0(B) and r_1(B) are stable

r_0(R) and r'_1(B) are stable

r_0(B) and r''_1(G) are stable

r_0(R) and r'''_1(G) are not!

Move to the other

stay

Possibility results

Two-robot Gathering problem on α-error compass

	SYm	CORDA
Semi-DC	impossible(α=π/4) possible(α<π/4)	open possible(α<π/6)
FiXedC	impossible(α=π/2)[1] →	← possible(α<π/2)

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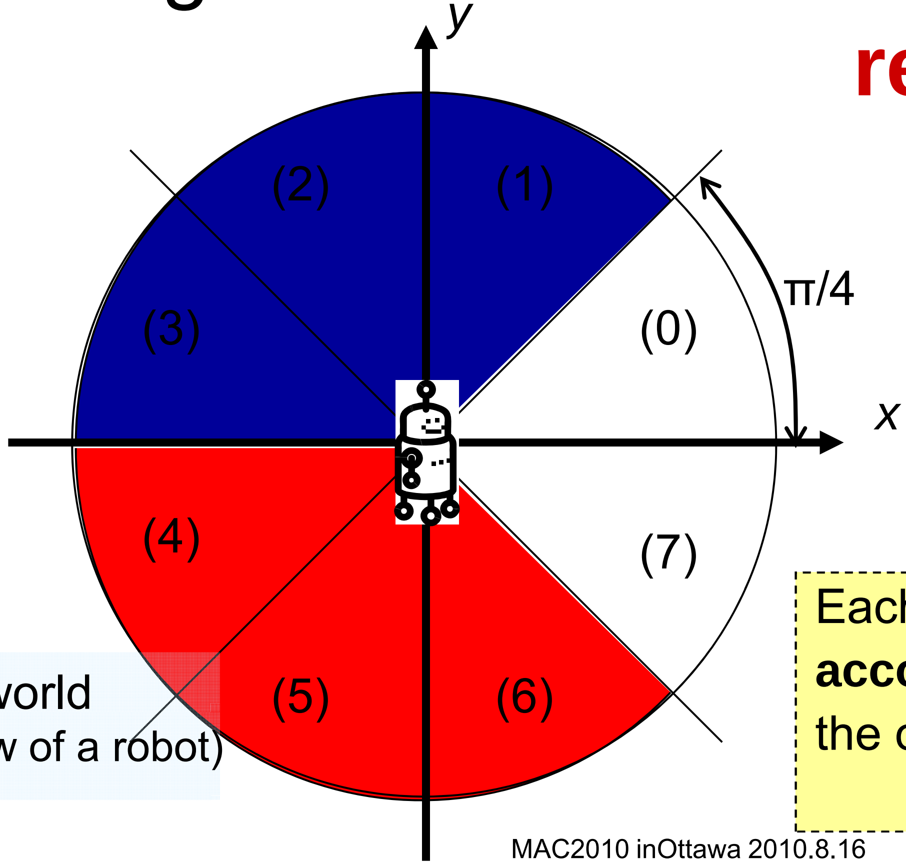
π/8-error SDC Algorithm on SYm

Point: How to decide the robots' behavior ?

Dividing the world (a view of a robot) into 8 sectors.

Coloring the divided world with three colors:

red, **blue** and **white**.



The world
(a view of a robot)

Each robot **decides its behavior according to the sector** in which the other robot is observed.

π/8-error SDC Algorithm on SYm

Algorithm

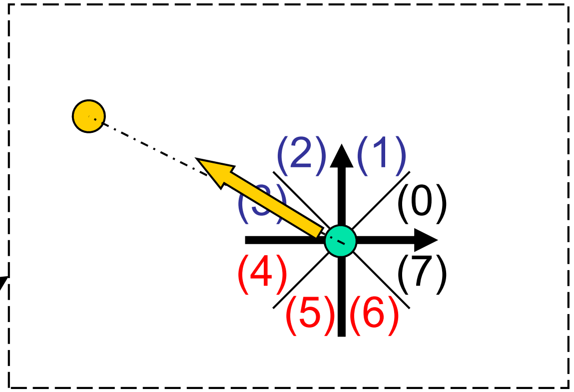
Result of observing the other robot

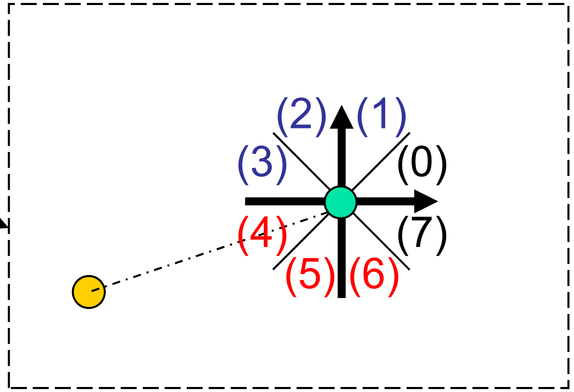
case: **no robot except me**
gathering is achieved

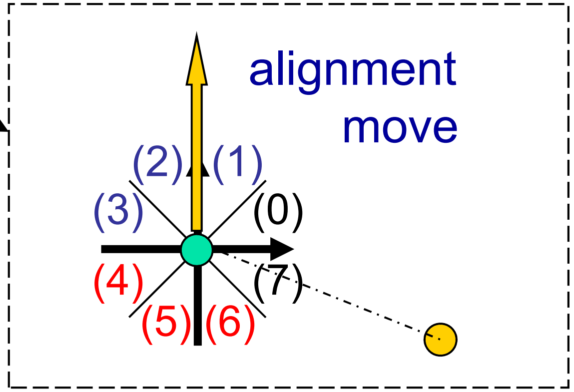
case: **in blue sectors (1), (2) or (3)**
move toward the other

case: **in red sectors (4), (5) or (6)**
no move

case: **in white sectors (7) or (0)**
move toward a right above point where I will be able to observe the other robot in the sector (6)



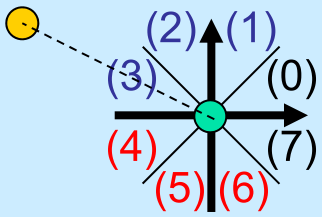




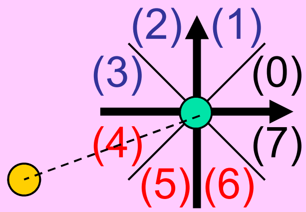
π/8-error SDC Algorithm on SYm

Why the robots can gather ?

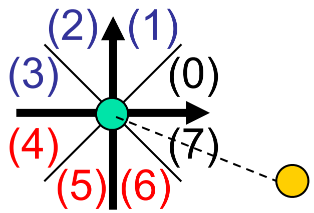
To show the correctness, three names of robots are introduced:



blue robot
observing the other robot in blue sector.



red robot
observing the other robot in red sector.



white robot
observing the other robot in white sector.

Dangerous Configurations

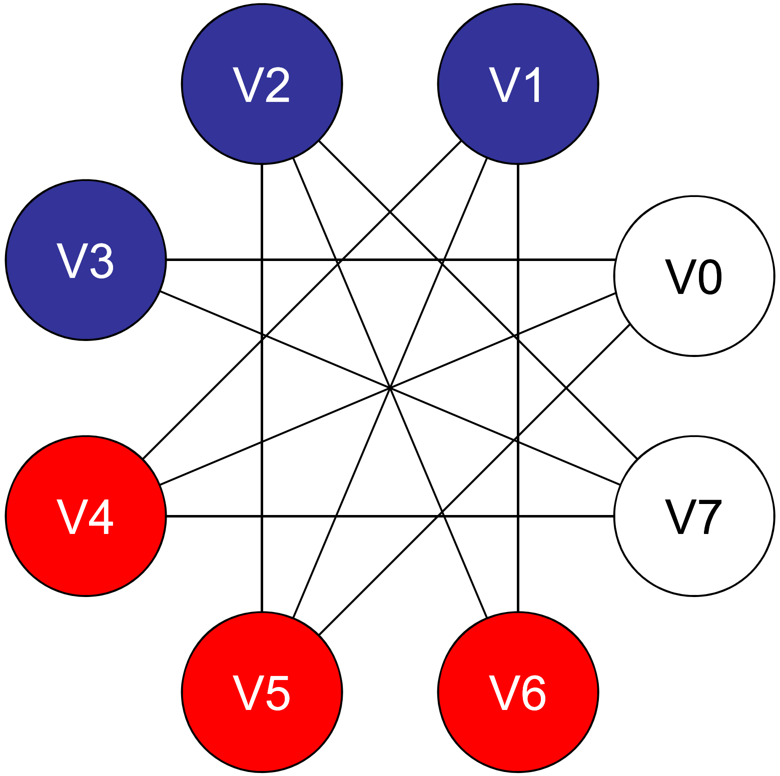
- **red - red** : deadlock!!
- **blue - blue** : swapping!! (loop)

We must show:

- **dangerous configurations never occur**
- **blue-red configuration is eventually reached**

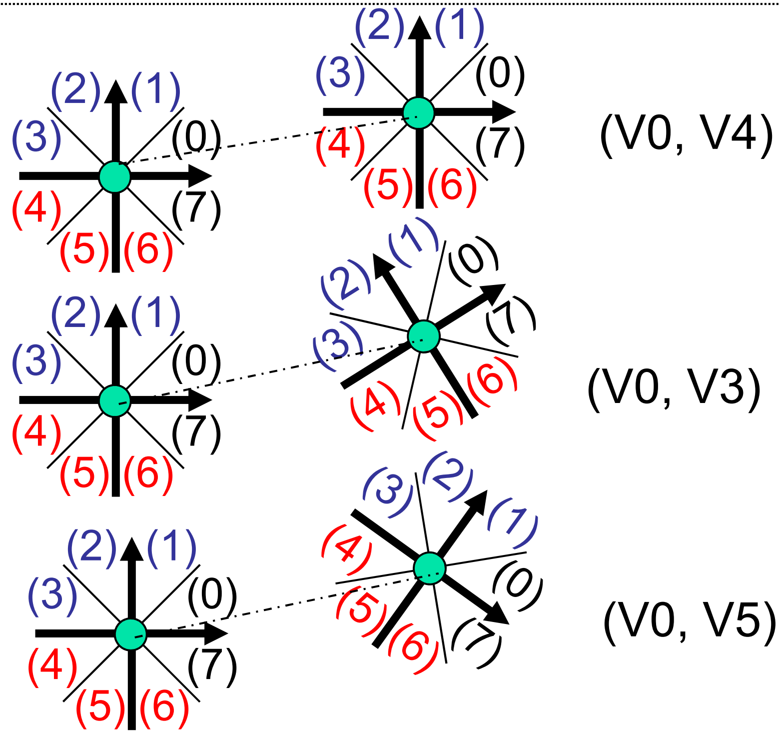
π/8-error SDC Algorithm on SYm

The Observation-Relation Graph



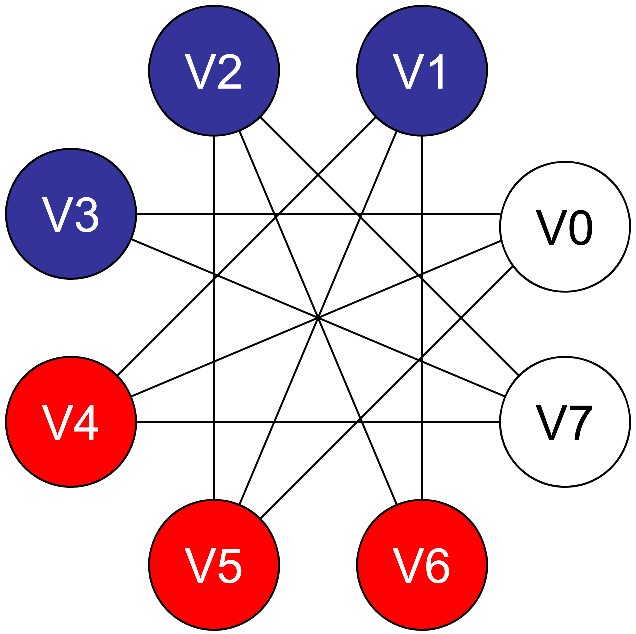
All nodes have three edges because of deviation of compass.

- **Vi** represents a robot who observes the other in **sector (i)**.
- An edge (**Vi**,**Vj**) represents that a configuration can exist such that robots observe each other in **sector (i)** and **(j),** respectively.



π/8-error SDC Algorithm on SYm

- Dangerous Configurations never occur
  - From the observation-relation graph with our sectoring and coloring, we know "red-red / blue-blue configurations never occur through executions."



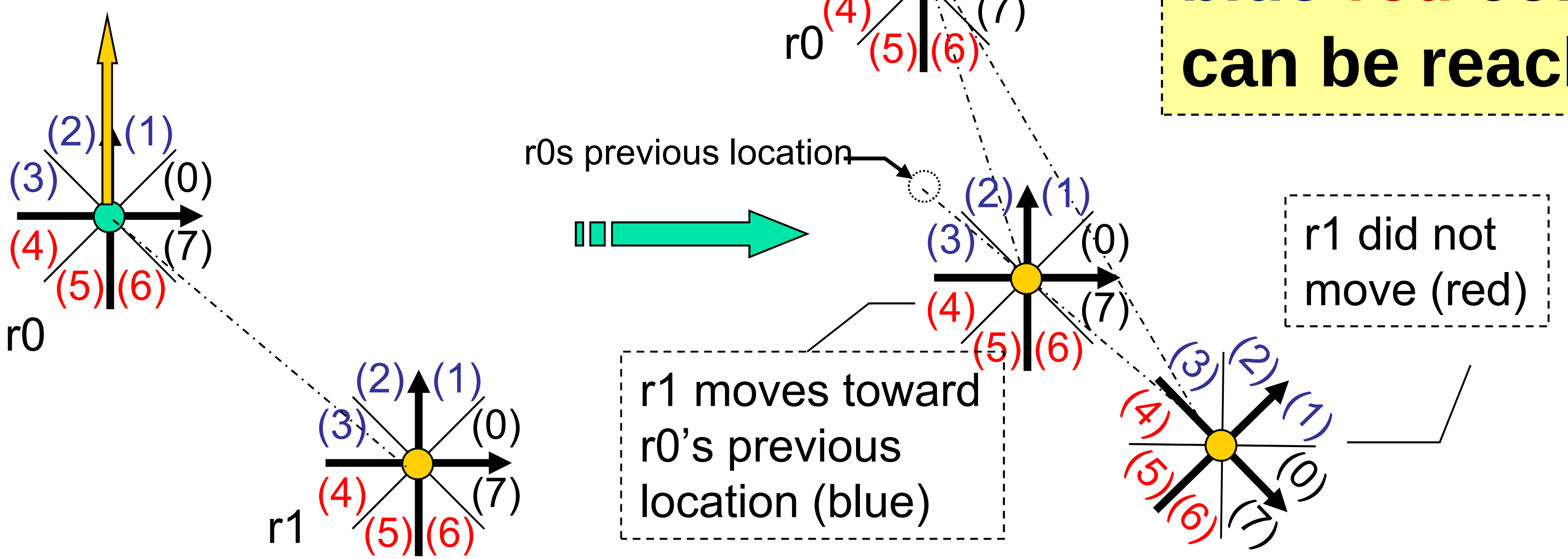
Only
blue-red, blue-white, red-white
configurations can exist.

π/8-error SDC Algorithm on SYm

- Blue and Red configuration will be eventually reached.
  - We need to show
    "From blue-white/red-white configuration, if r0 moves right above, **r0 can always observe r1 in**

We need to consider
two cases : r1 is "red" or "blue"

In both case,
blue-red conf.
can be reached.



π/8-error SDC Algorithm on SYm

- This algorithm can behave on CORDA
- The difficulty of proof on CORDA
  - Some robot r_0 observes r_1, r_1 may be moving

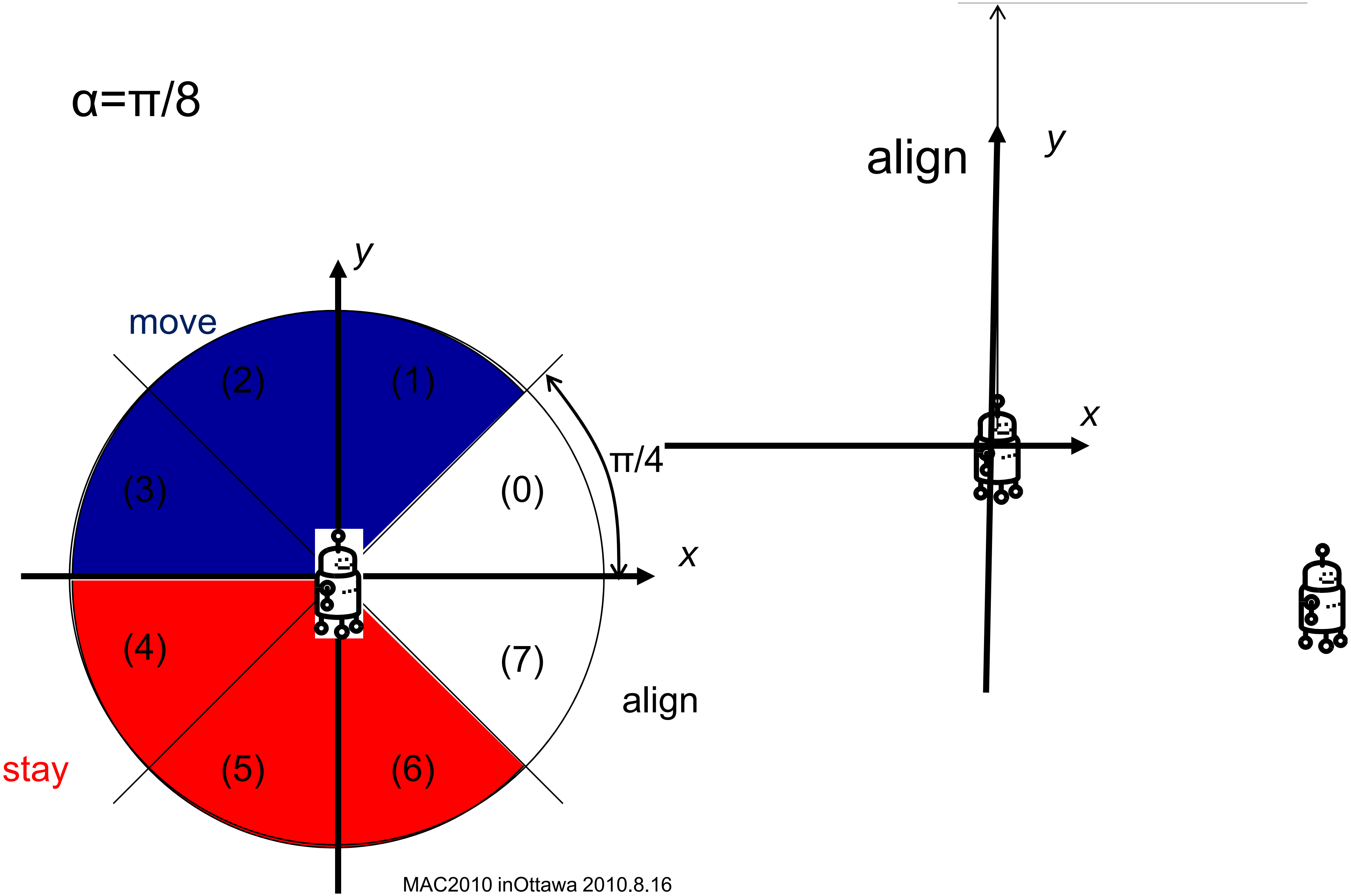
  → The relation when r_1 stops is different from the relation when r0 observed.

  (In SYm, such situation can not occur.)
  - Fast robot and very slow robot

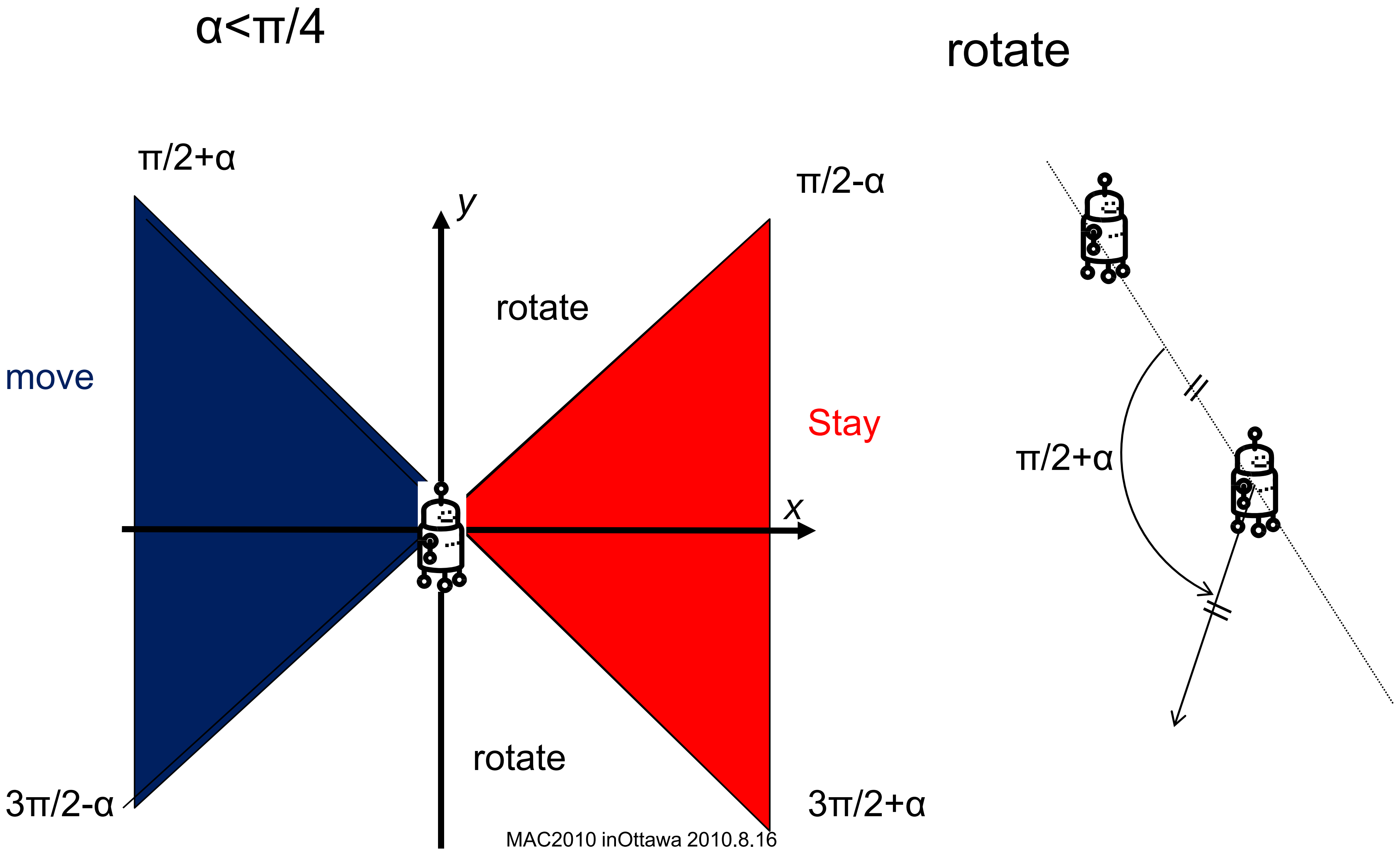
  →Most problems do not occur for 2 robots

π/8-error SDC Algorithm on CORDA(SYm)

$\alpha = \pi/8$



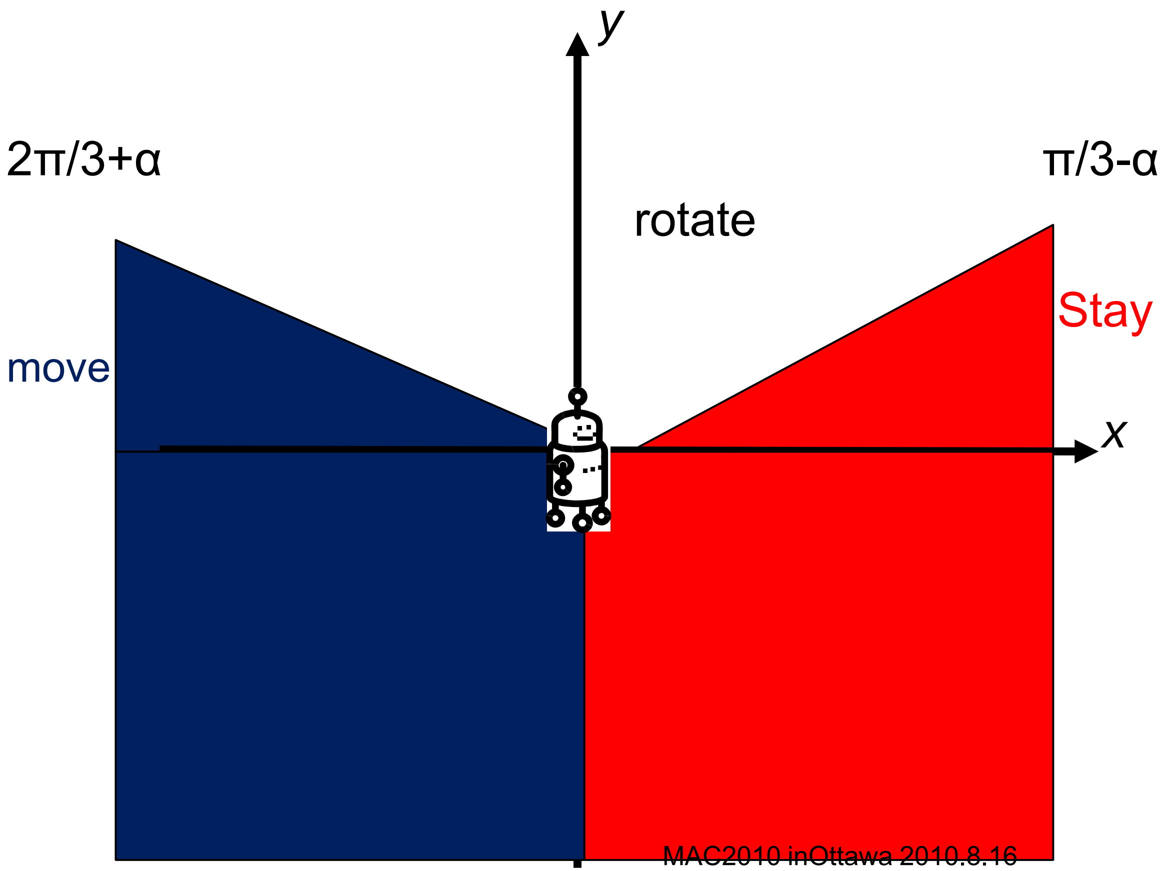
α-error SDC Algorithm on SYm

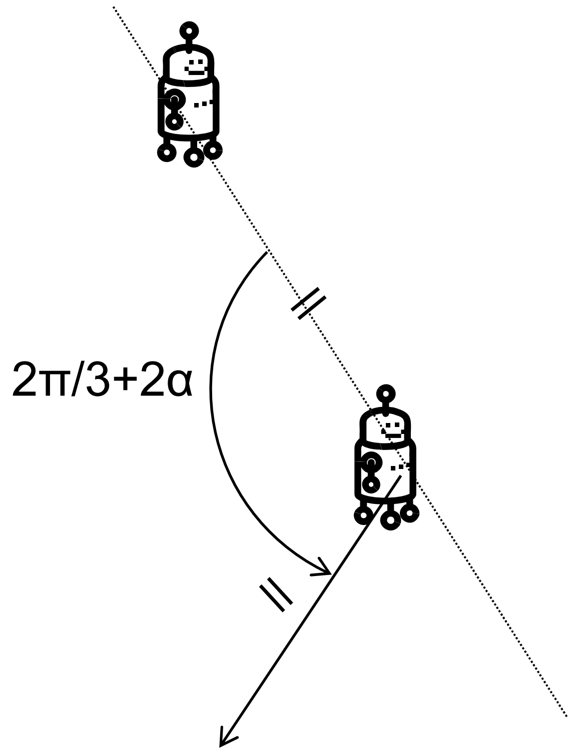


α-error SDC Algorithm on CORDA

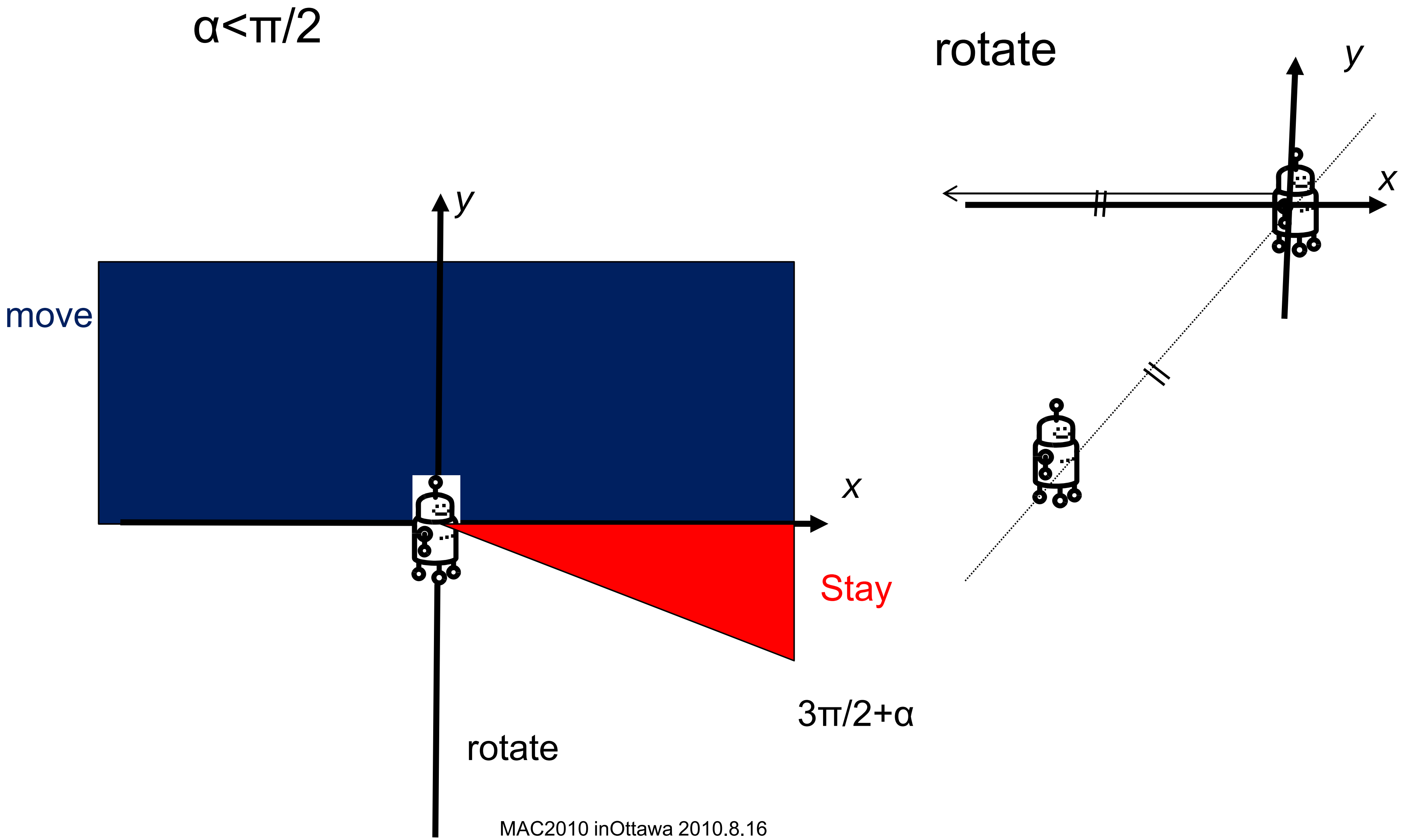
$\alpha < \pi/3$

rotate





α-error FXC Algorithm on CORDA(SYm)



Conclusions

Two-robot Gathering problem on α-error compass

	SYm	CORDA
Semi-DC	impossible($\alpha=\pi/4$) possible($\alpha<\pi/4$)	open possible($\alpha<\pi/6$)
FiXedC	impossible($\alpha=\pi/2$)[1] →	← possible($\alpha<\pi/2$)

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Open Problems

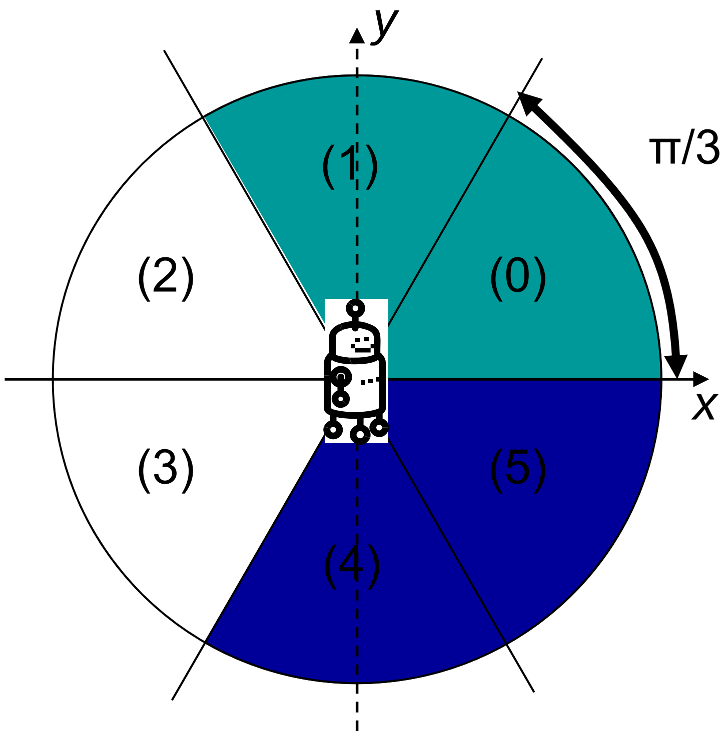
- Angle gap of SDC on CORDA
  - Impossible for $\alpha<\pi/6$ on CORDA
  - Possible for $\pi/4 >\alpha>\pi/6$ on CORDA
- Extension to n-robot system
  - SDC($\alpha<\pi/4$) on SYm is possible [DISC2007]

Thank you!

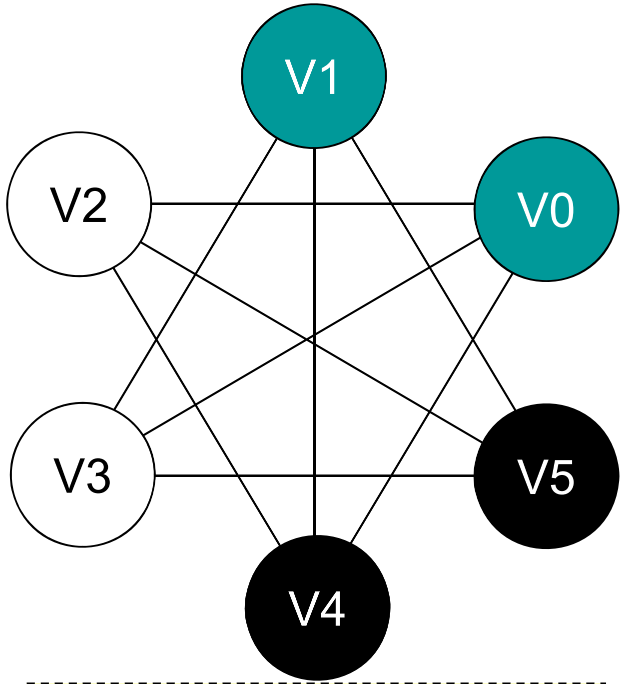
π/3-relative error FXC Algorithm

Basic idea is
same with π/4-absolute error SDC algorithm

Dividing the world into 6 sectors

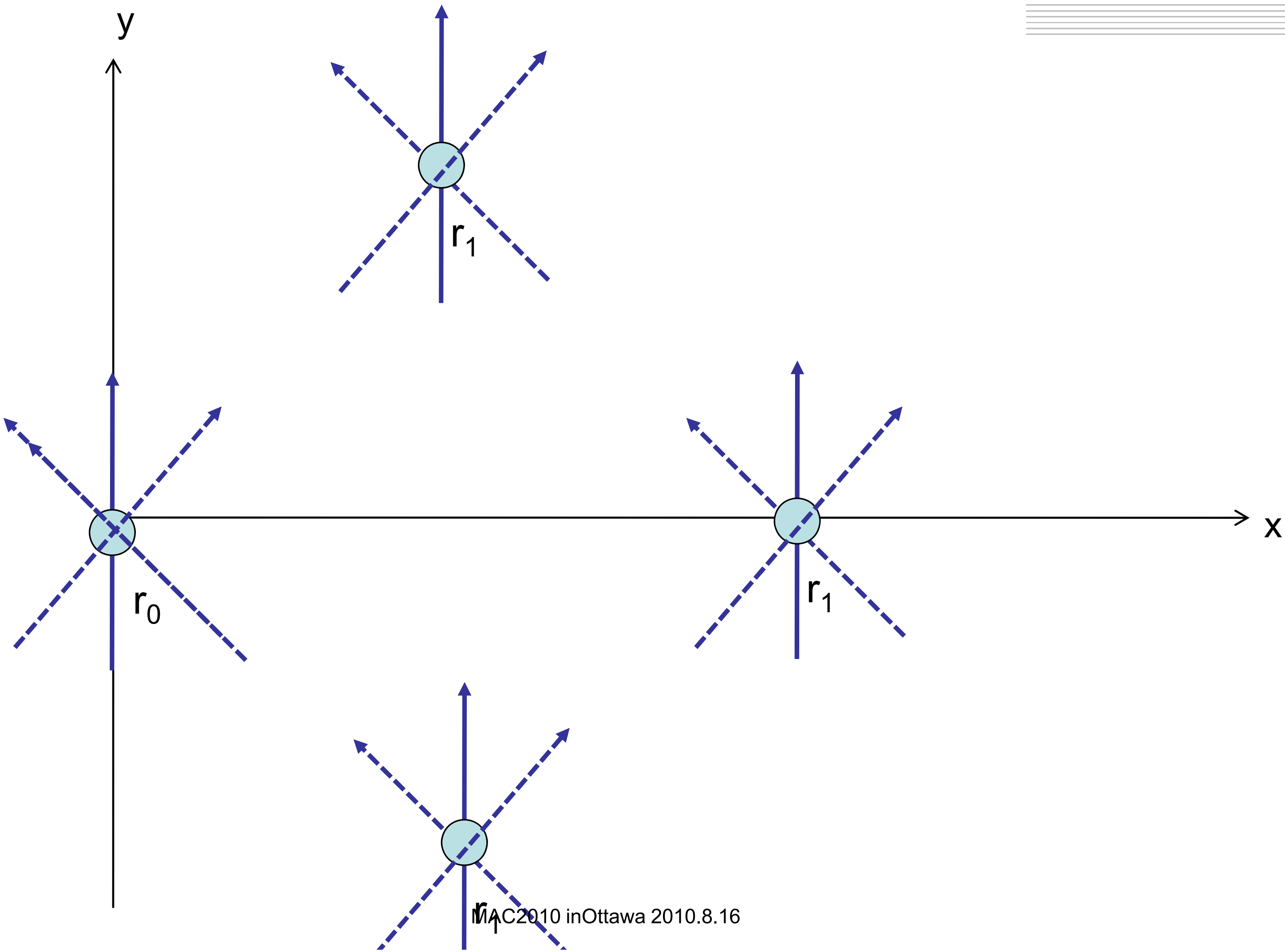


The observation-relation graph



no red-red, blue-blue

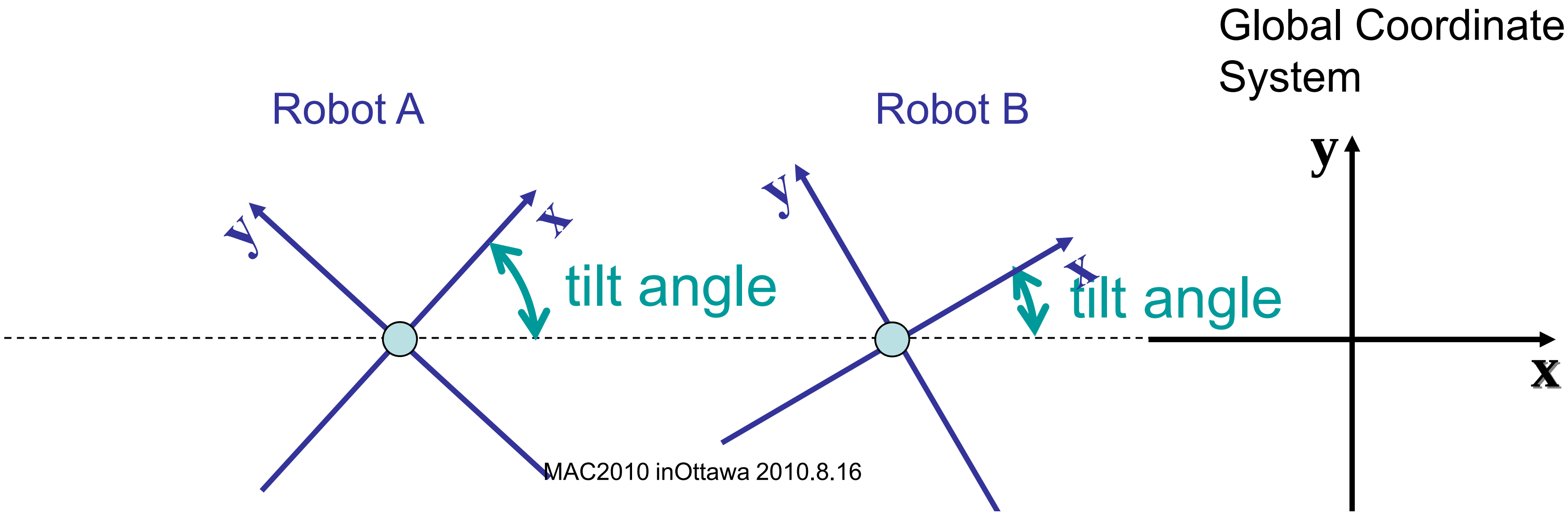
Impossibility(π/4-error compass , SDC and Sym)



Weakly-agreed compass

- Measuring Compass Agreement Level by tilt angle [Imazu et al., 05][Souissi et al., 06]
  - Tilt angle = ∠ formed by the global and local axis

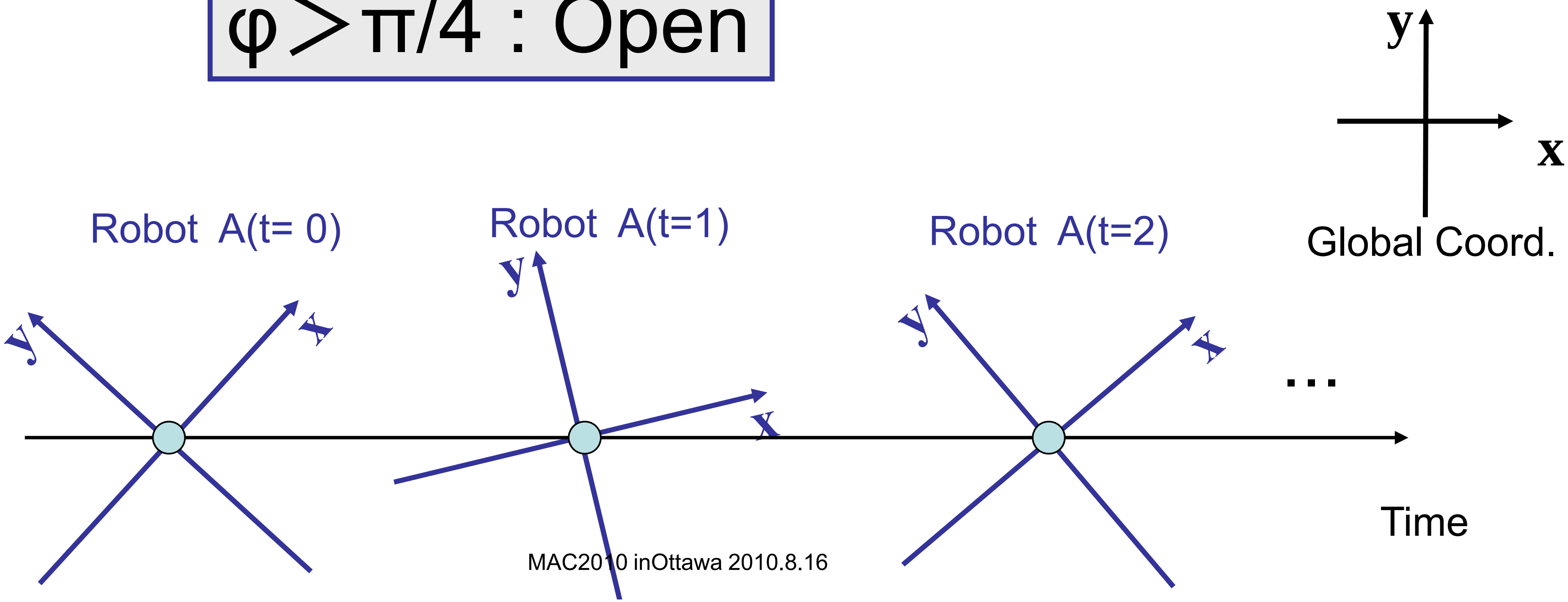
- Tilt angle of every robot < π :Solvable[Yamashita et al., 07]
  - Asynchronous / #robots = 2



Dynamic Compass

- Tilt Angle varies with time[Katayama et al., 07]
  - At the beginning of each cycle
  - Bounded by φ
- <u>φ≦π/4 : Solvable [Katayama.et.al, 07]</u>
  - Asynchronous / #robots = 2

φ＞π/4 : Open



Our Contribution

- Dynamic compass with $\varphi \leqq \pi/2-\varepsilon$: Solvable
  - ε (>0): Arbitrary small constant
  - Semi-synchronous / #robots n is arbitrary

  (The first result considering any #robots with disagreed compasses)

- $\varphi \geqq \pi/2$: Unsolvable
  - Semi-synchronous / #robots = 2

⇒ Our Result is optimal in terms of maximum tilt angle

Algorithm Design

- Algorithm for 2 robots with $\varphi = \pi/2-\varepsilon$

↓ Modify

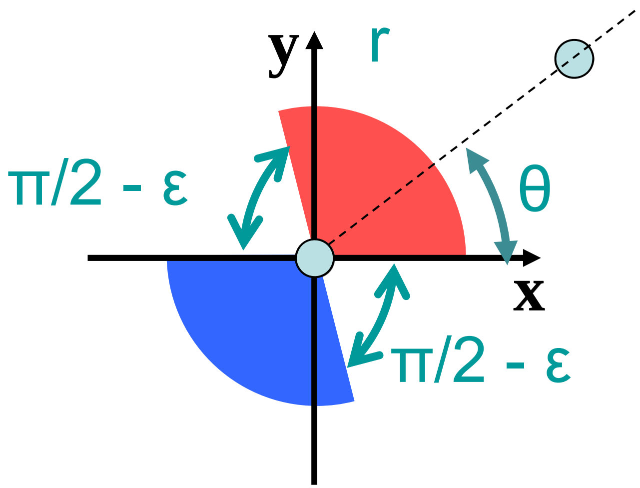
- "Conditional" Algorithm for n robots
  - Working correctly if the initial configuration has a unique Longest Distance Segment(LDS)
- LDS election algorithm
  - Starting any configuration,
    terminate a configuration with unique LDS

Composition

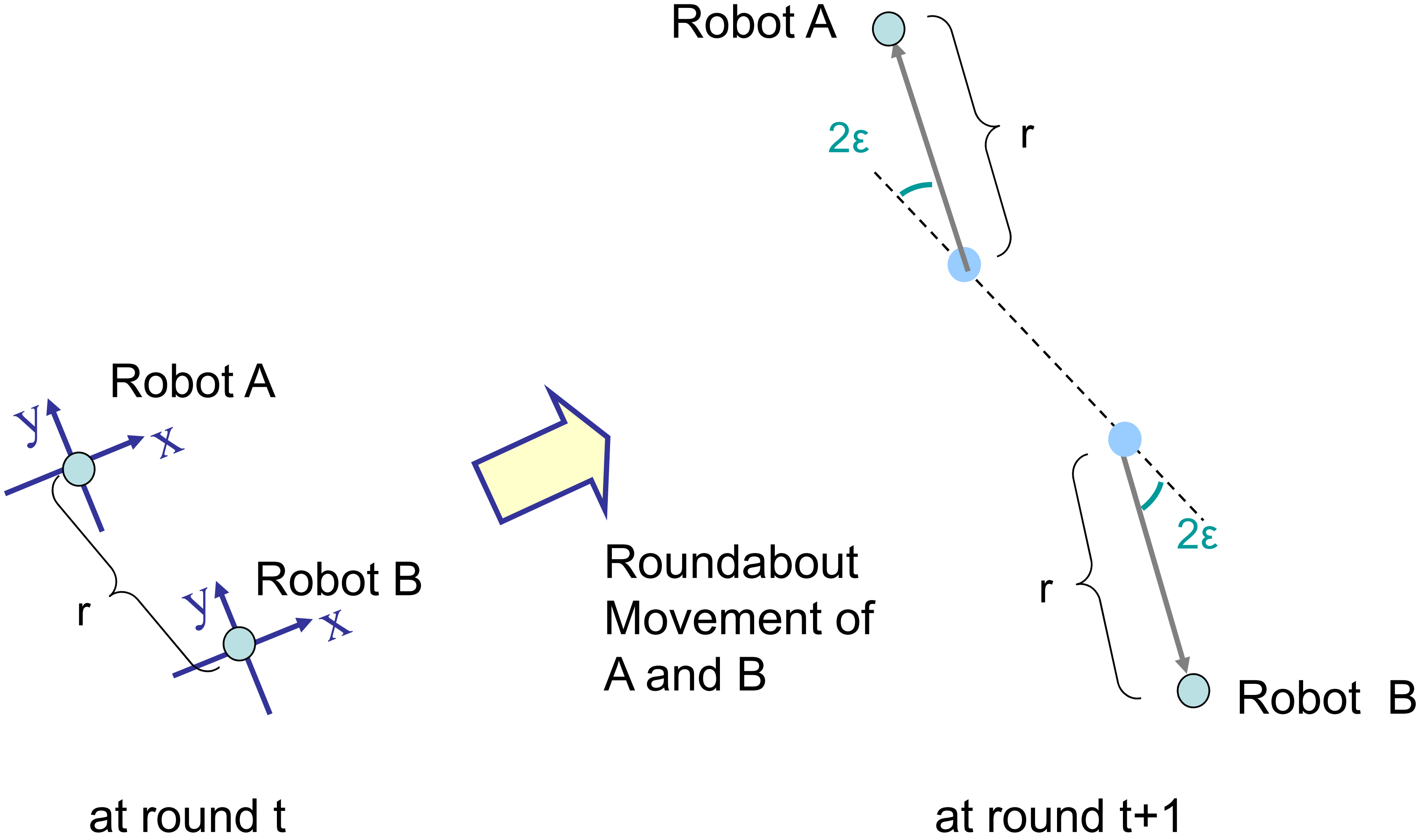
n-robot gathering algorithm

2-robot Algorithm

- Consists of three types of movement
  - θ : the angle at which a robot sees its partner
  - r : distance between two robots
    (in terms of observer's local coordinate sys.)
  - $0 \leqq \theta < \pi/2+\varepsilon$: Wait
    - No movement
  - $\pi \leqq \theta < 3\pi/2+\varepsilon$: Approach
    - Move to the partner's location
  - Otherwise : Roundabout
    - Move toward the angle $\theta+\pi-2\varepsilon$ with distance r



Roundabout Movement

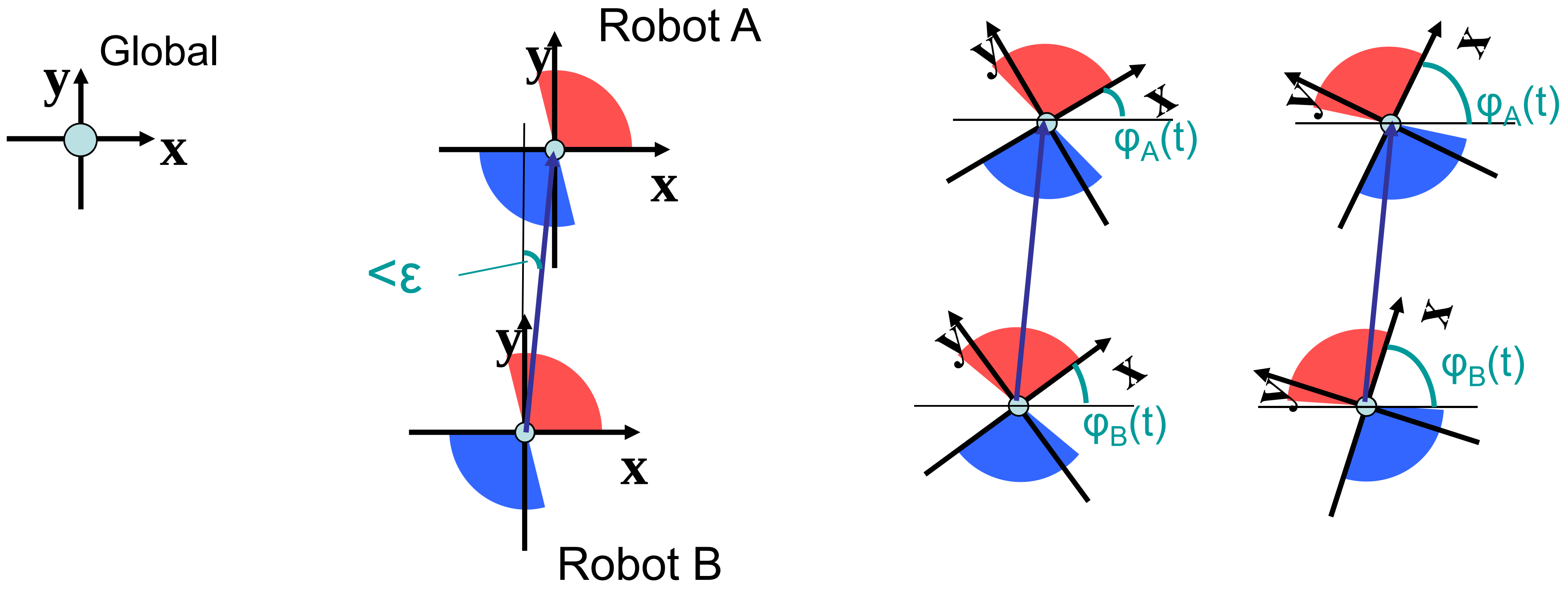


Correctness (1/5)

- Lemma 1
  - ∠ABy = ∠ formed by AB and the global y-axis < ε

  → Wait-Approach Relation is guaranteed

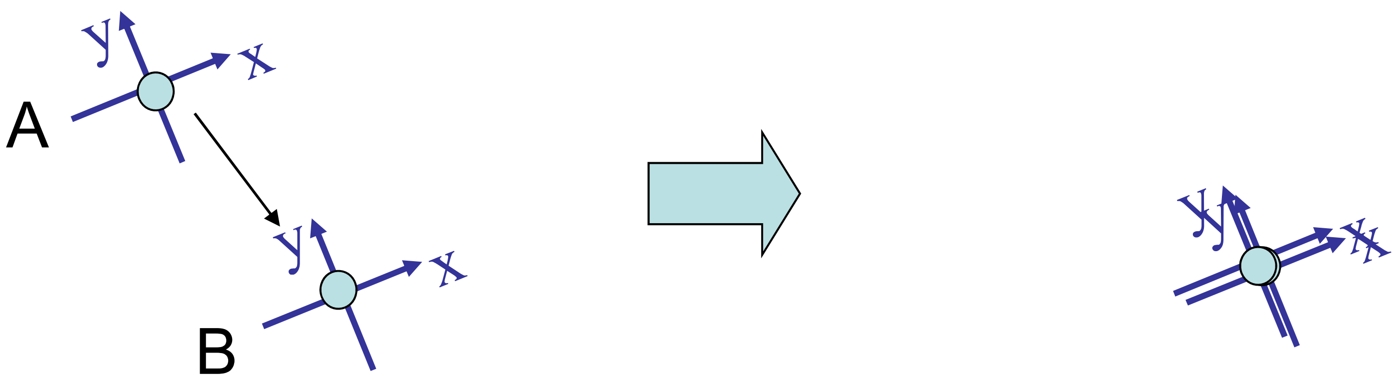
  (regardless of tilt angles of robot A and B)



Correctness(2/5)

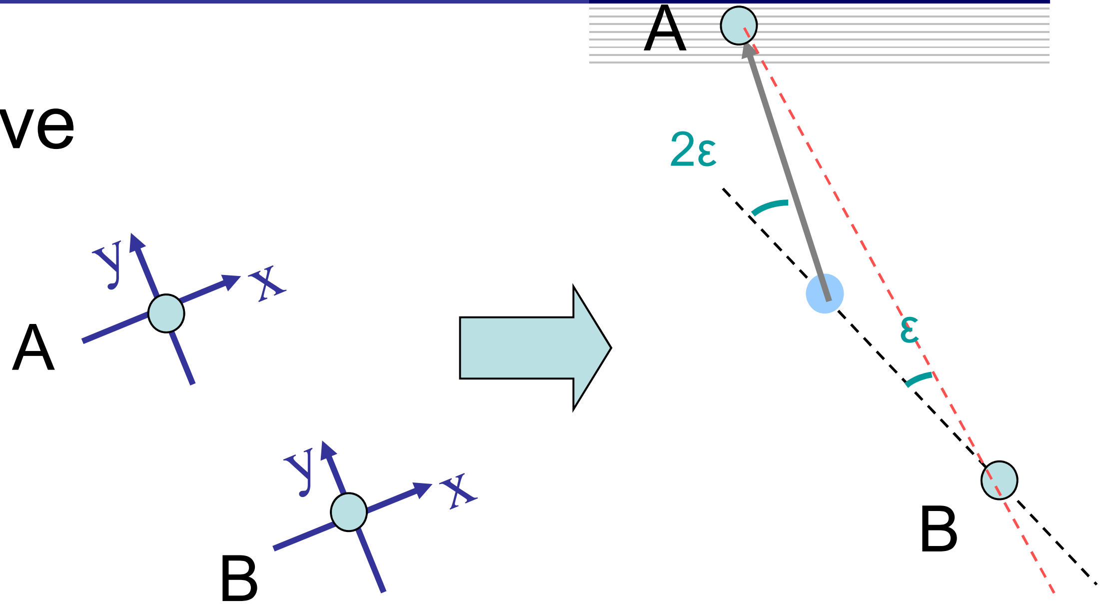
- Lemma 2
  - At any round, ∠ABy decreases by ε〜2ε unless gathering is achieved

- A: Approach move
- B: No movement (Wait or inactive)

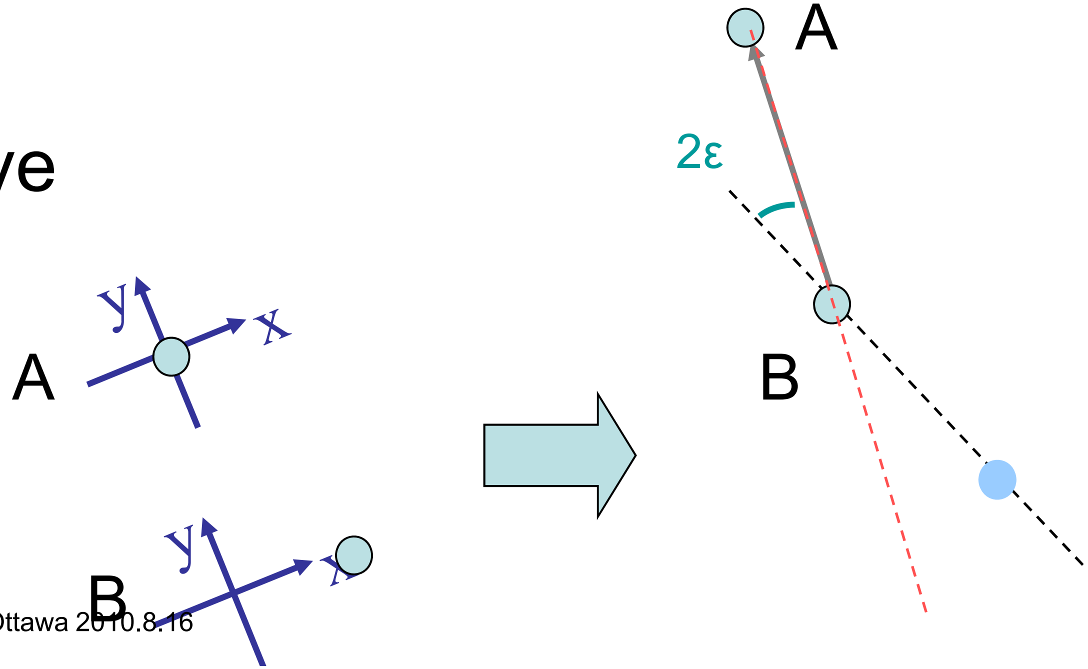


Correctness(3/5)

- A: Roundabout Move
  B: No movement
  → decrease by ε

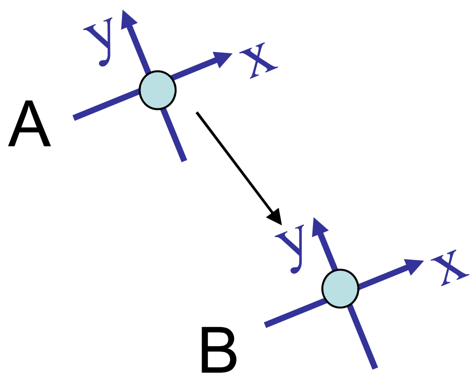


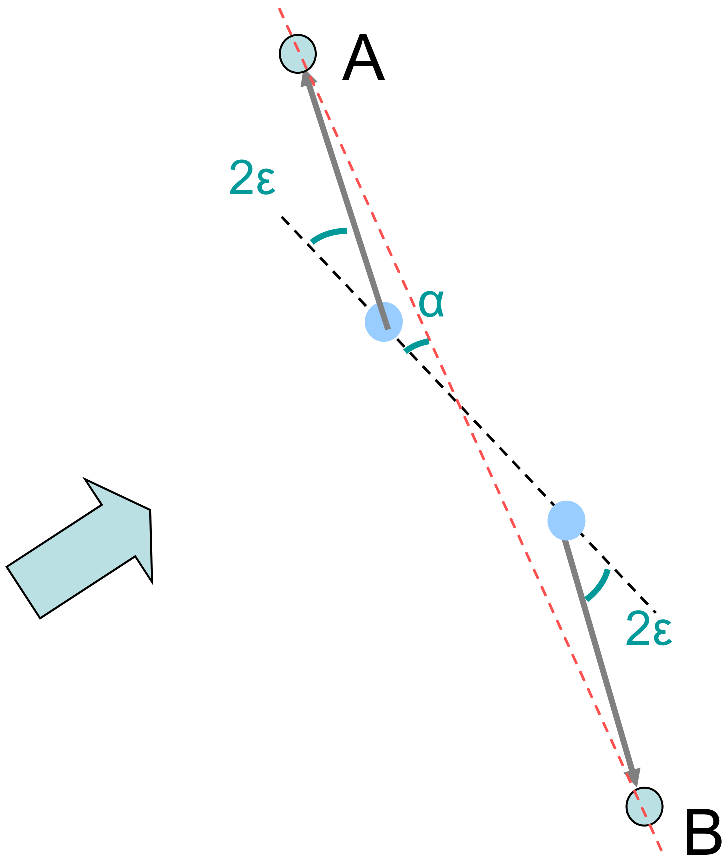
- A: Roundabout Move
  B: Approach Move
  → decrease by 2ε



Correctness (4/5)

- A: Roundabout move
  B: Roundabout move
  → decrease by ε～ 2ε





Correctness (5/5)

- From Lemma 2,

  $-\varepsilon \leqq \angle ABy < +\varepsilon$ eventually holds

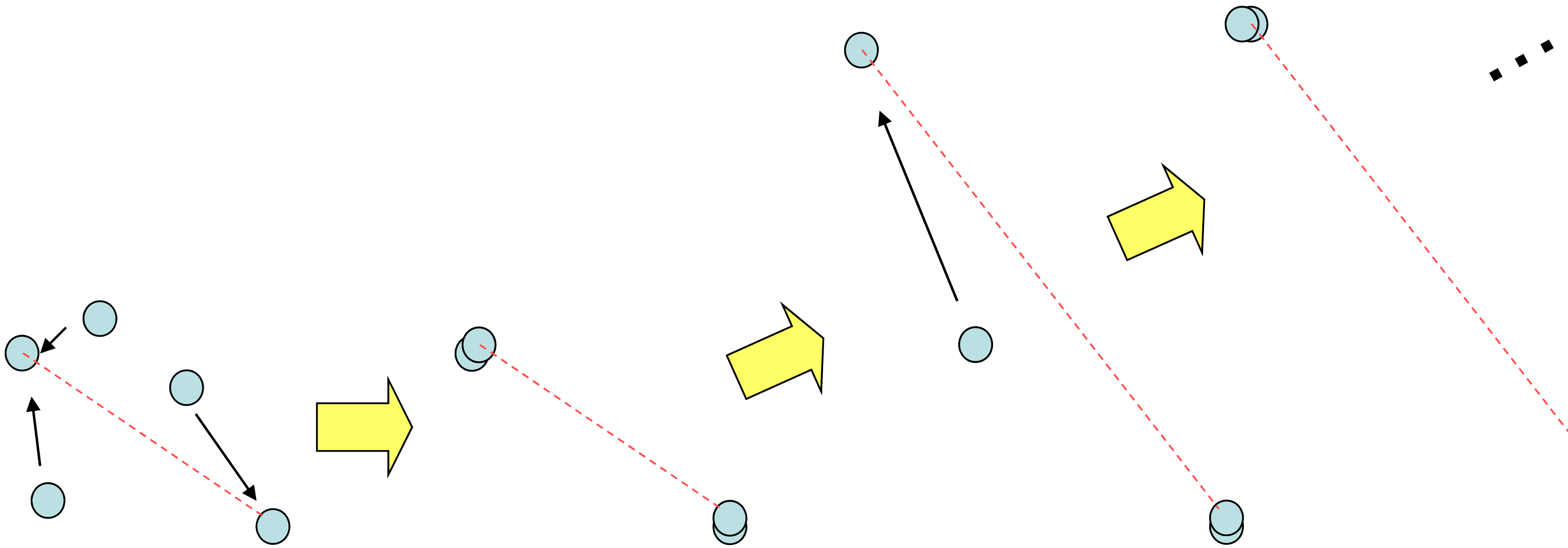
- From Lemma 1,

  If $-\varepsilon \leqq \angle ABy < +\varepsilon -\varepsilon$ holds, one robot approaches and the other waits.

Gathered !

n-robot algorithm under unique LDS(1/2)

- Robots are located at two points
  →All robots execute the two-robot algorithm
- Robots are located at more than two points
  →All robots move to one of two endpoints of LDS



Correctness of Conditional n-robot Alg.

- Lemma 3
  - ∠LDSy = ∠ formed by LDS and the global y-axis < ε

  → Wait-Approach Relation is guaranteed

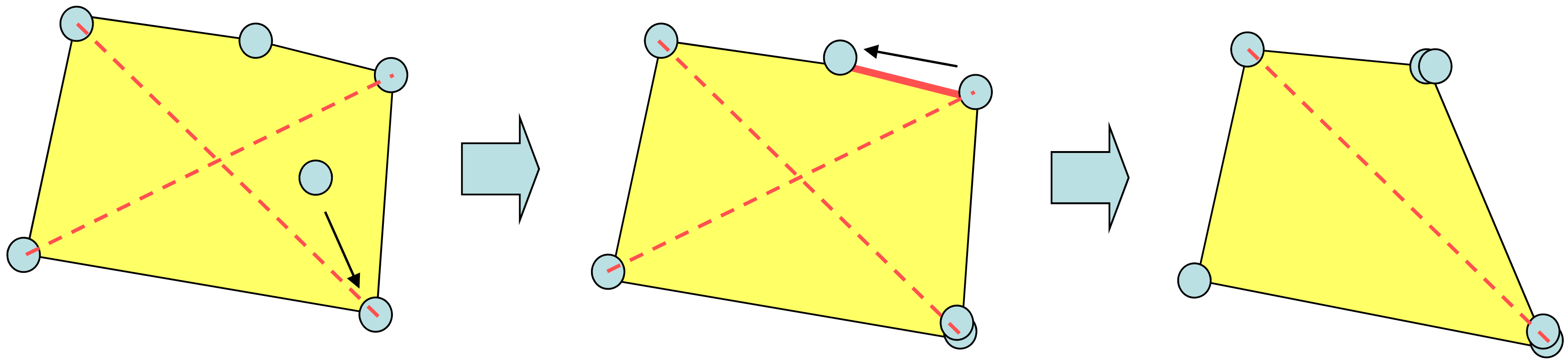
  (regardless of the title angle of each robots)

- Lemma 4
  - At any round, ∠LDSy decreases by ε～2ε

  unless gathering is achieved

Unique LDS Election (1/2)

- If two or more LDSs exist, each robot calculates the convex hull(CH)
  - Robots on the boundary : Wait
  - Inner robots : Moves to one of vertices
- Contracting the shortest edge of the CH



#edges of the CH decreases

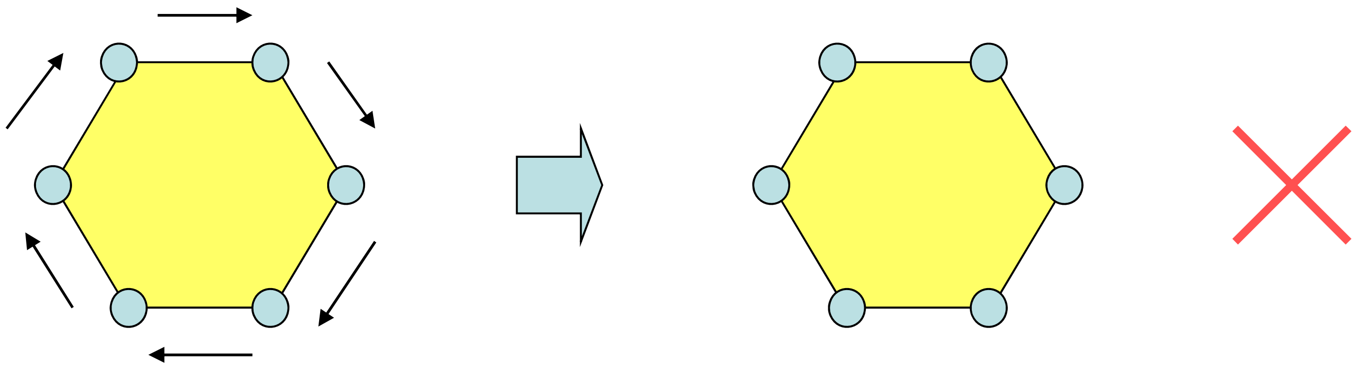
→Eventually unique LDS is elected (or gathered)

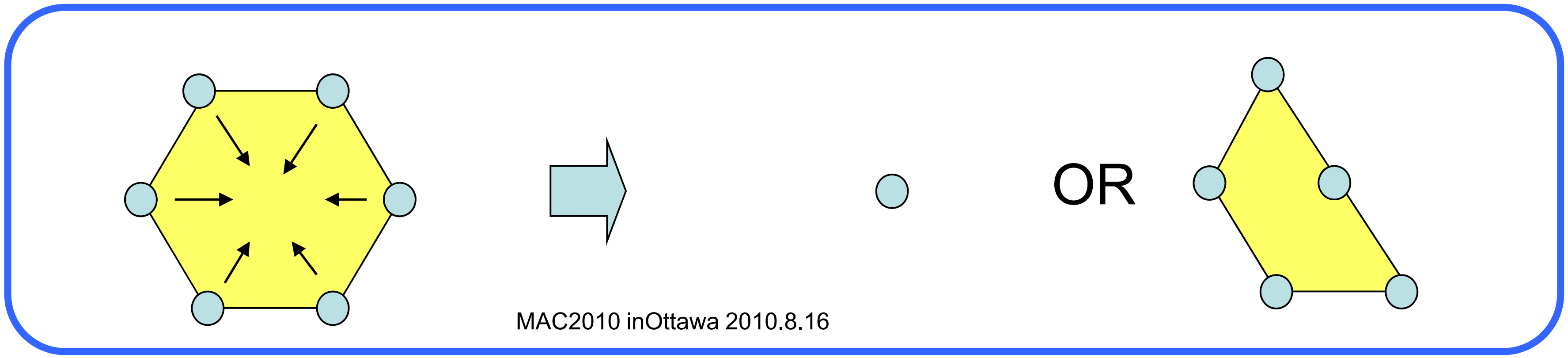
Unique LDS Election(2/2)

- If all edges have a same length

→ Robots moves to the center-of-gravity of the CH

  - All robots simultaneously move → gathered
  - A part of robots move → Symmetry is broken





Conclusion

- Gathering mobile robots with dynamic compasses
  - Tilt angle ≦ π/2-ε (Optimal)
  - Semi-synchronous model
  - Arbitrary #robots
- Open problem
  - Asynchronous model
    - π/2 < Maximum Tilt angle < π/4
    - Recently, two robots are solved for <π/3
    - #robots = 2, dynamic compass