

COMPUTATIONAL MOBILE ENTITIES

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DISCRETIZED CONTINUOUS SPACE


DISCRETIZED CONTINUOUS SPACE





| Practical Motivations <br> Very Few <br> Complex <br> Specialized <br> Robots |  |
| :--- | :--- |
|  | Many <br> - Easy to manufacture <br> - Cheap <br> - Easily replaceble <br> - Reusable |
|  | Micro-Robots <br> mac-2010 |
|  |  |



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System
In addition to sensing and computing the micro-robots are capable of locomotion
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Cooperation of autonomous mobile robots
    Robotics
    Artificial Intelligence
    Behaviour of Social Animals (Insects)
        Swarms and Ants
    Control
    Distributed Algorithms
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\section*{System}

Mobility adds useful system capabilities
- Patrol a wide area
- Re-positioning for better surveillance
- Work in hostile/dynamic environments
- Risky area surrounding or surveillance
- Search and rescue missions
- Space exploration
- Military operations mac-2010

\section*{Main Research Question}

Minimum level of capabilities
(sensorial, computational, and motorial) robots must possess to COLLECTIVELY
solve a given task


\section*{Basic Coordination Tasks}
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Gathering

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\section*{Basic Coordination Tasks}
\begin{tabular}{|c|c|c|}
\hline Gathering & - \({ }^{\circ}\) 。 & 88 \\
\hline Specific Patterns &  & : \(\because 0\). \\
\hline Alignment & & -0.0 \\
\hline
\end{tabular}


\section*{Complex Tasks}

\section*{Mine Sweeping}

Hazardous Retrieval

Rescue Operation





\section*{Visibility : Snapshot}
-Snapshot returns the coordinates of the other robots (seen as points) in terms of my local coodinate system


\section*{Visibility}
- Radius

Limited


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\section*{Crucial Factors}
- Visibility Limited \(\rightarrow\) Unlimited
- Memory

\section*{Memory}







\section*{Crucial Factors}
- Visibility Unimited \(\rightarrow\) Limited
- Memory Persistent \(\rightarrow\) Oblivious
- Agreement on local coordinate systems


Total Agreement: Direction and Orientation

The robots agree on a common direction and orientation of both axes


\section*{Agreement 1: \\ Direction and Orientation}

\section*{The robots agree on a common direction and} orientation of both axes


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Total Agreement: Direction and Orientation

Might disagree on origin and unit of length


Agreement 2: without Chirality

The robots agree on a common direction and
orientation of axis


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\section*{Agreement 2: without Chirality}

Might disagree on origin and unit of length might NOT share the same handedness


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Agreement 2: without Chirality

The robots agree on a common direction and orientation of axis


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\section*{Agreement 3: Direction}

The robots agree on direction of both axes


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\section*{Agreement 3: Direction}

The robots agree on direction of both axes


Agreement 4: Orientation
The robots agree on orientation of both axes


Agreement 3: Direction

Might disagree on origin and unit of length might disagree on orientation of axes


Agreement 4: Orientation

The robots agree on circular orientation of plane


Agreement 4: Orientation


Agreement 5:


Levels of Agreement on Coordinate System

\section*{Crucial Factors}
- Visibility Unimited \(\rightarrow\) Limited
- Memory Persistent \(\rightarrow\) Oblivious

Agreement on local coordinate systems


\section*{Crucial Factors}
- Visibility Unimited \(\rightarrow\) Limited
- Memory Persistent \(\rightarrow\) Oblivious
- Agreement on local coordinate systems
- Time/Synchronization

Time / Synchronization

There are three basic models

Fully synchronous (FSYNC)

Semi synchronous (SSYNC)

Asynchronous (ASYNC)

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Time: Fully Synchronous (FSYNC)

There is a global clock tick reaching all robots simultaneously
- At each clock tick every robot become active and perform its cycle atomically

Time: Fully Synchronous (FSYNC)


Time: Semi Synchronous (SSYNC)


Time: Semi Synchronous (SSYNC)

There is a global clock tick reaching all robots simultaneously

At each clock tick every robot is either active or inactive, and only active robots perform their cycle atomically.
[Suzuki Yamashita 96]

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Time: Semi Synchronous (SSYNC)

Fair Scheduler:
every robots becomes active infinitely often

Time: Asynchronous (ASYNC)

Time: Asynchronous (ASYNC)


Time: Asynchronous (ASYNC)
There is NO global clock and robots do not have a common notion of time

Each robot becomes active at unpredictable time instants

Each computation and movement takes a finite but unpredictable amount of time
[Flocchini Prencipe Santoro Widmayer 99]
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Time: Asynchronous (ASYNC)

COMPUTE


Time: Asynchronous (ASYNC)

COMPUTE

\section*{Computation based}
on obsolete information


Time: Asynchronous (ASYNC)


\section*{Crucial Factors}
- Visibility Unimited \(\rightarrow\) Limited
- Memory Persistent \(\rightarrow\) Oblivious
- Agreement Total \(\rightarrow\) No
- Time/Synchronization FSYNCH \(\rightarrow\) ASYNCH

\section*{Crucial Factors}
- Visibility Unimited \(\rightarrow\) Limited
- Memory Persistent \(\rightarrow\) Oblivious

Agreement Total \(\rightarrow\) No
- Time/Synchronization FSYNCH \(\rightarrow\) ASYNCH
- Mobility

\section*{Mobility 1:}

In the MOVE ...

\section*{Mobility :}

Executing the protocol with the state variables and the current view as input, gives as output a destination point \({ }^{\circ}\)


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\section*{Mobility 1: UNLIMITED MOTORIAL ENERGY}

In the MOVE the robot moves continuously until it reaches \(O\)



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\section*{Mobility 1: UNLIMITED MOTORIAL ENERGY}

In the MOVE the robot moves continuously until it reaches \(O\)



\section*{Mobility 2:}

In the MOVE ...


\section*{Mobility 2: LIMITED MOTORIAL ENERGY}

In the MOVE the robot moves continuously until it reaches \({ }^{\circ}\) or has traveled a distance \(D\).


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\section*{Mobility 3:}

In the MOVE ...

Mobility 2: LIMITED MOTORIAL ENERGY

In the MOVE the robot moves continuously until it reaches \({ }^{\circ}\) or has traveled a distance \(D\).


\section*{Mobility 3: UNPREDICTABLE DISTANCE}

In the MOVE the robot can stop anytime.
If it does not reach \({ }^{\circ}\), it moves at least \(d\).


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\section*{Mobility 3: UNPREDICTABLE DISTANCE}

In the MOVE the robot can stop anytime.
If it does not reach \({ }^{\circ}\), it moves at least \(d\).


\section*{Mobility 3: UNPREDICTABLE DISTANCE}

In the MOVE the robot can stop anytime.
If it does not reach 0 , it moves a least \(d\).


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\section*{Mobility 3: UNPREDICTABLE DISTANCE}

In the MOVE the robot can stop anytime.
If it does not reach \({ }^{\circ}\), it moves at least \(d\).


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\section*{HOMOGENEOUS}

\section*{every robot executes the same protocol}
all robots are identical and indistingushable

\section*{TERMINATION}

\section*{NON-OBLIVIOUS}
within finite time, every robot enters a terminal state

OBLIVIOUS
within finite time, every robot performs only null actions

\section*{IMPOSSIBILITY}

No deterministic protocol exists which always correctly solves the problem terminating in finite time


\section*{Pattern Formation}


\section*{Pattern Formation}

Pattern \(P\) is a set of (distinct) coordinates in the plane.
Number of robots is equal to the number of coordinates. At the beginning,
the robots are in distinct arbitrary positions.
At the end,
the positions of the robots must "coincide" with \(P\) (translation, scaling or rotation).

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\section*{Computation/Protocol}

FORMATION
the pattern is formed within finite time (terminating protocol)

\section*{CONVERGENCE}
the pattern is never formed but the robots' position converges to the pattern in the limit
(non-terminating protocol)
(terminating approximate solution)
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\section*{Arbitrary Pattern Formation}

\section*{Arbitrary Pattern Formation}

The n robots must form
any input pattern \(P\) of \(n\) points
starting from any initial position

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\section*{Leader Election Problem \\ Leader Election Problem}

After a finite number of cycles, all the robot deterministically agree on (choose) the same robot \(L\), called the leader

Theorem
For \(n>2\), if Arbitrary Pattern Formation is solvable with level of agreement \(A\) then
the Leader Election problem is solvable with level of agreement \(A\)

\section*{Leader Election Problem}

After a finite number of cycles, all the robot deterministically agree on (choose) the same robot \(L\), called the leader


APF : Agreement and Computability


Agreement on Coordinate System (e.g., compass, GPS)
even if the robots are asynchronous and oblivious

\section*{Leader Election Problem}

After a finite number of cycles, all the robot deterministically agree on (choose) the same robot \(L\), called the leader


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APF: Total Agreement

\section*{Theorem.}

With a common coordinate system, a set of asynchronous anonymous oblivious robots can form any input pattern in finite time.

APF : Agreement and Computability


APF: Agreement and Computability



No Chirality

APF : Agreement and Computability

even if asynchronous and oblivious
[Flocchini Prencipe Santoro Widmayer 02]

Theorem. If the number of robots is even, the Leader Election problem is unsolvable without chirality.

Even if they have unlimited persistent memory And are fully synchronous. [FPSW 99]
arbitrary pattern formation is impossible

\section*{APF : Agreement and Computability}


Theorem. If the number of robots is even, the Leader Election problem is unsolvable without chirality.

Even if they have unlimited persistent memory And are fully synchronous. [FPSW 99]

arbitrary pattern
formation is impossible

Theorem. If the number of robots is even, the Leader Election problem is unsolvable without chirality.

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arbitrary pattern formation is impossible

Pattern Formation : \(n\) even and no chirality

Q: Which patterns can be formed when \(n\) is even and no chirality?
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APF : Agreement and Computability

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# ALWAYS

\longrightarrow x

```
\begin{tabular}{c}
\(y\) \\
\(\uparrow\) \\
NES if \(n\) odd \\
NO \(n\) even
\end{tabular}

Pattern Formation : n even and no chirality

Q: Which patterns can be formed when \(n\) is even and no chirality?

A: NO asymmetric patterns

\section*{even if}
- unbounded persistent memory
- fully synchronous system

\section*{NO ASYMMETRIC PATTERNS}

By contradiction: A lets the robots form asymmetric patterns, starting from an arbitrary initial configuration

Let us execute \(A\) to form \(P\) (asymmetric)



\section*{NO ASYMMETRIC PATTERNS}

Case 2


Final Configuration (asymmetric)
June 13, 2007 MAC - 2010

\section*{NO ASYMMETRIC PATTERNS}

\section*{Case 2}

Contradiction!!

Final Configuration (asymmetric)
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Pattern Formation : \(n\) even and no chirality

Q: Which patterns can be formed when \(n\) is even and no chirality?

A: symmetric patterns ....


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\section*{Pattern Formation : \(n\) even and no chirality}

Q: Which patterns can be formed when \(n\) is even and no chirality?
A: symmetric patterns with >0 empty axis of symmetry.

Yes:


No:


Pattern Formation : n even and no chirality

A: symmetric patterns with >0 empty axis of symmetry.

Local operations:
- Compute an empty axis of symmetry \(S\) (the same for all robots) of input pattern \(P\)

Rotate \(P\) so that \(S\) is parallel to the agreed orientation of \(Y\)






Robots on K:
- Do not move as long as there are robots inside Left/Right not on Final Positions (free robots)
|| If no free robot is inside Left/Right, the topmost on K moves towards one of the Final Positions still available

Robots inside Left/Right:
\| The free robot in my side closest to a Final Position \(p\) still available in my side, moves towards \(p\)
- If there are no Final Positions available in my side, go on K

\section*{Robots on K:}

Do not move as long as there are robots inside Left/Right not on Final Positions (free robots)

If no free robot is inside Left/Right, the topmost on \(K\) moves towards one of empty Final Positions

Robots inside Left/Right:
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- Do not move as long as there are robots inside Left/Right not on Final Positions (free robots)

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If there are no Final Positions available in my side, go on K



APF : Agreement and Computability

(no chirality)

APF : Agreement and Computability
even if
- unbounded persistent memory
- fully synchronous system


IMPOSSIBLE

No agreement
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APF : Agreement and Computability

\(?\)
No agreement
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any totally synchronous execution will transform the \(n-G O N\) into another \(n-G O N\)


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\section*{APF : NO AGREEMENT}
even if
- unbounded persistent memory
- fully synchronous system

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APF: NO AGREEMENT

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\section*{corollary}

The only patterns that could be formed without agreement are n-GONS


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Uniform Circle Formation


Pattern Formation : no agreement

Q: can uniform circles be formed when there is no agreement?

Pattern Formation : no agreement

Q: can n-GONs be formed when there is no agreement?

\section*{Pattern Formation : no agreement}

Q: can uniform circles be formed when there is no agreement?

YES with unlimited persistent memory in SSYNCH [Yamashita and Suzuki, 1996]

\section*{Pattern Formation : no agreement}

\section*{Q: can uniform circles be formed when there is no agreement?}

YES with unlimited persistent memory in SSYNCH
[Yamashita and Suzuki, 1996]

CONVERGENCE oblivious [Defago Konagaya, 2002]
[Chatzigiannakis et al., 2004] [Defago Souissi, 2008] SPECIAL SIZES oblivious [Dieudonne and Petit, 2007]

\section*{Pattern Formation : no agreement}

YES with unlimited persistent memory in SSYNCH
[Yamashita and Suzuki, 1996]
YES obliviously in SSYNCH
[Yamashita and Suzuki, 2008]
[Petit and Dieudonne, 2009]

\section*{Pattern Formation : no agreement}

Q: can uniform circles be formed when there is no agreement?

YES with unlimited persistent memory in SSYNCH
[Yamashita and Suzuki, 1996]
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\section*{Pattern Formation : no agreement}
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PAT[OBLIVIOUS] = PAT[NON-OBLIVIOUS]

```

YES with unlimited persistent memory in SSYNCH
[Yamashita and Suzuki, 1996]
YES obliviously in SSYNCH
[Yamashita and Suzuki, 2008]
[Petit and Dieudonne, 2009]
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Pattern Formation : no agreement
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    PAT[OBLIVIOUS] = PAT[NON-OBLIVIOUS]
    ```
\(=\) PAT [OBLIVIOUS] \(=\) PAT[NON-OBLIVIOUS]
            OBLIVIOUSNESS
    IS NOT A HANDICAP
        IN FSYNCH AND SSYNCH
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Pattern Formation : no agreement

WHAT ABOUT IN ASYNCH?
PAT [OBLIVIOUS] ? = ? PAT[NON-OBLIVIOUS] ASYNCH ASYNCH

CAN OBLIVIOUS ROBOTS FORM A UNIFORM CIRCLE ASYNCHRONOUSLY?

Pattern Formation : no agreement

\section*{WHAT ABOUT IN ASYNCH?}

OBLIVIOUSNESS IS NOT A HANDICAP IN FSYNCH AND SSYNCH

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Pattern Formation : no agreement

Some claims
[Katreniak 2005] [Petit Dieudonne, 2009]

CAN OBLIVIOUS ROBOTS FORM A UNIFORM CIRCLE ASYNCHRONOUSLY?

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