

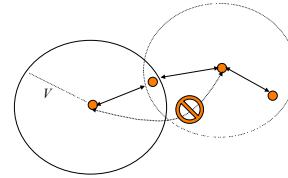
## Distributed Coordination of a Set of Autonomous Mobile Robots: The Unlimited Visibility Setting

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Università di Pisa  
Italy



MAC - 15<sup>th</sup> August 2010

## Radius of Visibility: Limited / Unlimited



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## Timing Models



**ASync/CORDA** - Fully asynchronous  
[Flocchini et. Al, 1999]

Arbitrary & varying operation rates and delays

**SSync/SYM** - Semi-synchronous [Suzuki  
+Yamashita, 1996]

Fixed time cycles, but robots may be active / inactive

**FSync/SYM** - Fully synchronous [Suzuki  
+Yamashita, 1996]

Fixed time cycles, all robots active in every cycle



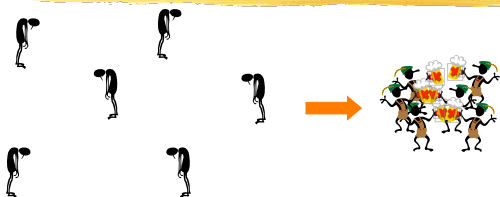
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## Problem 1: Gathering, (Aggregation, rendez-vous, homing)

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## Gathering - Unlimited Visibility



Initially the robots are in arbitrary distinct positions.

In finite time, they **gather** in the same place.

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## Motivations

A conference is a **gathering** of important people  
who singly can do nothing, but together  
can decide that nothing can be done

Fred Allen (1894-1956)

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## Gathering

- In spite of its apparent simplicity, this problem has been tackled in several studies

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## Gathering

### Unlimited Visibility - SSYNC

Ando, Oasa, Suzuki, Yamashita  
Siam Journal Of Computing, 1999

- Instantaneous activities
- $n=2$ , the problem is **unsolvable**


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## Gathering, $n=2$

### Unlimited Visibility - SSYNC

In fact, since the robots have **no dimension...**

 ...and cannot **bump...**

...moving them towards each other is not useful...

...but it works if they can **bump!**

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## Gathering

### Unlimited Visibility - SSYNC

Ando, Oasa, Suzuki, Yamashita  
Siam Journal Of Computing, 1999

- Instantaneous activities
- $n=2$ , the problem is **unsolvable**
- $n>2$ , they provide an obvious algorithm that let the robots gather in **finite time**

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## Gathering

### Unlimited Visibility - SSYNC

- MAIN IDEA:** Starting from **distinct initial** positions, we move the robots in such a way that eventually there will be **exactly** one position that two or more robots occupy

- Each time a robots becomes active, it recognizes the configuration of the robots (7 possible configurations)

- It moves accordingly

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## Gathering

### Unlimited Visibility - SSYNC

#### Few examples:

- Let  $C$  be the **smallest enclosing circle** of the observed configuration
- If  $n > 3$ , and there is exactly one robot  $r$  inside  $C$ 
  - $r$  moves towards one of the robots on the rim of  $C$ ; the others do not move
- If  $n > 3$ , and there are **more than two robots inside  $C$** 
  - The robots inside  $C$  move towards the center of  $C$ , while the other do not move (the center of  $C$  does not change)

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## Gathering

### Unlimited Visibility - SSYNC

- If  $n > 3$ , and all robots are on the rim of  $C$ 
  - All robots move towards the center of  $C$
- In the last case, at the next time instant
  - Either **some** robots are inside  $C$  and some on the rim of  $C$ 
    - One of the previous cases applies
  - Or **all** robots are again on the rim of  $C$ 
    - This same case applies again

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## Gathering, ASYNC

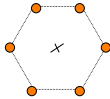
- In ASYNC, several factors render this problem difficult to solve
- Major problems arise from symmetric configurations....

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## Difficulties

If at the beginning....

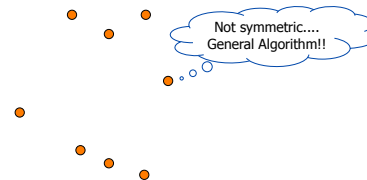


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## Difficulties

If at the beginning....



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## Difficulties

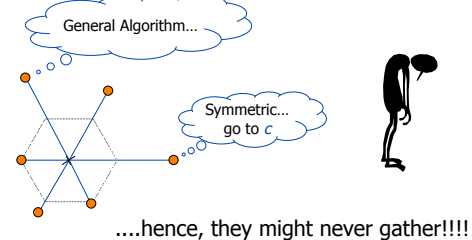


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## Difficulties

### Symmetric



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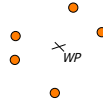
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## Gathering—easy solution

Easy Solution: **Weber Point (Weiszfeld, '36)!**

Given  $r_1, \dots, r_n$ :  $WP = \arg \min_{p \in \mathbb{R}^2} \sum_i \text{dist}(p, r_i)$

1. It is unique (Weiszfeld, '36)



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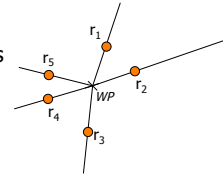
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2. WP is Weber Point of points on  $[r_i, WP]$  (Weiszfeld, '36)



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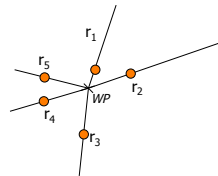
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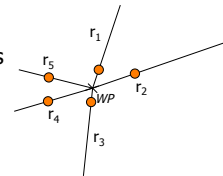
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## Gathering—easy solution

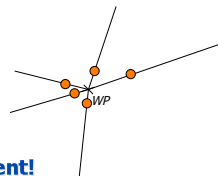
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Given  $r_1, \dots, r_n$ :  $WP = \arg \min_{p \in \mathbb{R}^2} \sum_i \text{dist}(p, r_i)$

1. It is unique (Weiszfeld, '36)
2. WP is Weber Point of points on  $[r_i, WP]$  (Weiszfeld, '36)



**WP Invariant Under Movement!**



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## Gathering—easy solution

Easy Solution: **Weber Point (Weiszfeld, '36)!**

Algorithm:

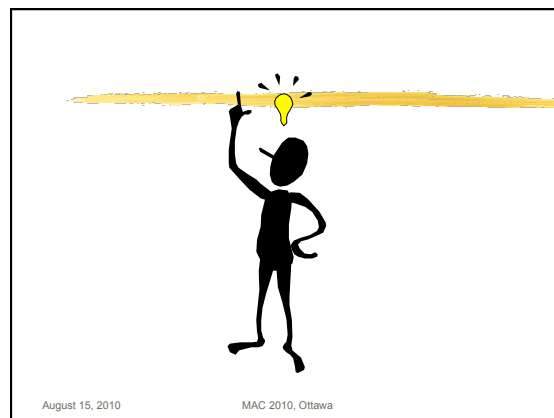
1. Compute **WP**
2. Move Towards **WP**

**Unfortunately, WP is not computable!**

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## Gathering

(Unlimited Visibility, no agreement)

$n=2$ : **Unsolvable** (by Suzuki *et al.*),  
unless they can **bump** into each other!

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## Gathering

(Unlimited Visibility, no agreement)

$n=2$ : Unsolvale (by Suzuki *et al.*),  
unless they can **bump** into each other!

$n=3, 4$ : **Always solvable!**

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## Gathering, $n=3$

(Unlimited Visibility, no agreement)

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## Gathering, $n=3$

(Unlimited Visibility, no agreement)

Properties of  $c_e$ :

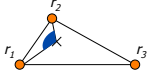
$c_e$  = center of equiangularity

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### Gathering, $n=3$

(Unlimited Visibility, no agreement)



Properties of  $c_e$ :

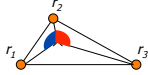
- ❖  $r_1 \hat{c}_e r_2$

$c_e$ =center of equiangularity

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### Gathering, $n=3$

(Unlimited Visibility, no agreement)



Properties of  $c_e$ :

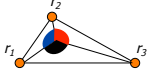
- ❖  $r_1 \hat{c}_e r_2 = r_2 \hat{c}_e r_3$

$c_e$ =center of equiangularity

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### Gathering, $n=3$

(Unlimited Visibility, no agreement)



Properties of  $c_e$ :

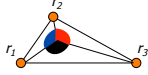
- ❖  $r_1 \hat{c}_e r_2 = r_2 \hat{c}_e r_3 = r_3 \hat{c}_e r_1$

$c_e$ =center of equiangularity

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### Gathering, $n=3$

(Unlimited Visibility, no agreement)



Properties of  $c_e$ :


- ❖  $r_1 \hat{c}_e r_2 = r_2 \hat{c}_e r_3 = r_3 \hat{c}_e r_1$
- ❖ Invariant under movement

$c_e$ =center of equiangularity

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### Gathering, $n=3$

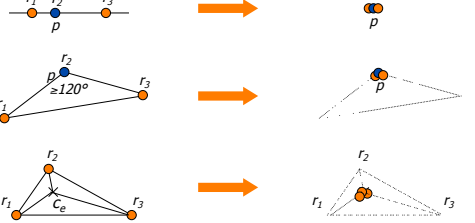
(Unlimited Visibility, no agreement)



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### Gathering, $n=3$

(Unlimited Visibility, no agreement)



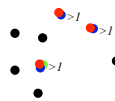
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## General Schema

### ■ Use of Multiplicity Detection

■  $n=3,4$  (and even with the use of Weber Point)

*Is there 1 or more than 1 robot at a point?*



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## General Schema

The general idea of the solutions is based on **multiplicity detection**, as follows

1. At the beginning, robots on distinct positions
2. Get a scenario where there is only one point  $p$  with multiplicity greater than one
3. All robots move towards  $p$

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## Multiplicity Detection

If the robots **cannot** detect **multiplicities**...



*is as....*



...the proposed solutions do not work!!!!

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## Multiplicity Detection

For  $n=2$ , the problem is **not solvable** (Suzuki et al., 1999)!

It is possible to design an **adversary** that lets the robots occupy **two** distinct positions on the plane in a finite number of cycles...



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## Multiplicity Detection

*....hence....*

Problem not solvable with  $n=2$



No multiplicity detection



**Problem not solvable for any  $n$ !**

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## Gathering

**No agreement** on the local coordinate systems, and **oblivious** robots....



**[TCS 2007]**

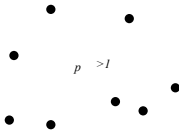
**Necessary Condition: Multiplicity Detection!**  
(in SSYNC, hence in ASYNC)

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## The Main Idea (ICALP 2003)

- All distinct positions in initial configuration
- Some robots gather at a point  $p$
- Point  $p$  is unique point with multiplicity  $>1$
- All robots move to  $p$

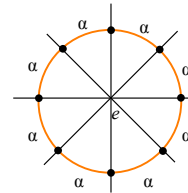


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## A Special Case

The robots can be in a totally symmetric configuration.

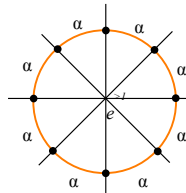


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## Equiangular Configurations

- All adjacent robots have angle  $\alpha$  w.r.t. center  $e$
- Center  $e$  is easy to compute
- Move all robots towards  $e$



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## Algorithm Skeleton

**If** strict multiplicity: move there

**Else if** equiangular: move to center

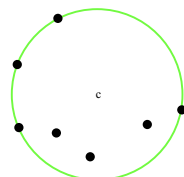
**Else** elect some robots to gather

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## Smallest Enclosing Circle

- Unique
- Easy to compute
- Invariant if suitable robots move

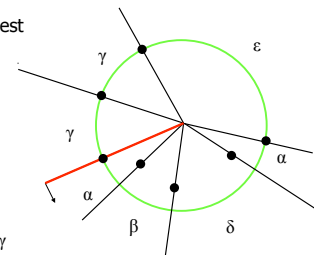


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## String of Angles

- W.r.t. center of smallest enclosing circle
- Circular string
- Can contain 0's



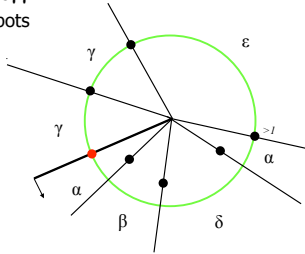
Lex. minimum:  $\alpha\beta\delta\alpha\epsilon\gamma\gamma$

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## 1 Starting Position

- Lex. min string:  $\alpha\beta\delta\alpha\epsilon\gamma\gamma$
- Unique ordering of robots

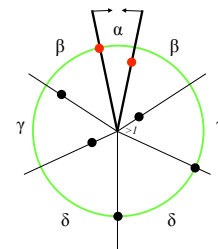


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## 2 Starting Positions

- Palindrome:  $\alpha\beta\gamma\delta\delta\gamma\beta$



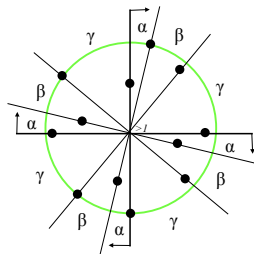
- or Periodic:  $\alpha\beta\gamma\delta\alpha\beta\gamma\delta$

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## 3..n-1 Starting Positions

- Periodic:  $\alpha\beta\gamma\alpha\beta\gamma\dots$



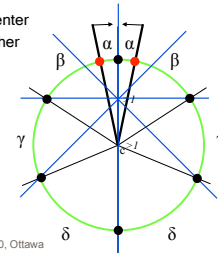
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## Problem: Case Switches

The algorithm:

- If strict multiplicity: move there
- Else If equiangular: move to center
- Else elect some robots to gather

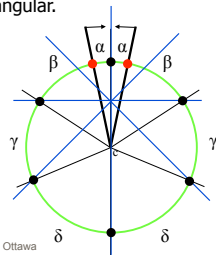


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## Cautious Movements

The moving robots stop at any **critical point** where the configuration becomes equiangular.

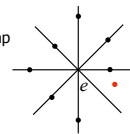


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## Finding Critical Points

- 1 starting position:  
is there center of equiangularity with 1 gap
- 2 starting positions:  
same with 2 gaps
- 3..n-1 starting positions:  
cannot generate equiangular configurations



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## Other Studies on Gathering

- Cieliebak, LATIN 2004
  - Non-oblivious robots, ASYNC
  - No multiplicity detection
- Gathering via Center of Gravity
  - Peleg et al.
- Gathering with faulty robots

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## Other Studies on Gathering

- Gathering with dynamic compasses
  - Katayama et al., SemiSync
- Gathering with limited visibility
  - Flocchini et al., TCS 2005, ASYNC

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## Problem 1.a: Gathering via CoG

Peleg et al.

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## Center-of-Gravity (CoG) algorithms

### General approach:

In each cycle, the robots:

1. calculate some **median** position of the group
2. move towards that position

### Natural variant:

Use the **Center-of-Gravity (CoG)**

(a.k.a. **center of mass** / **barycenter**)

of the robot group  $\vec{c}[t] = \frac{1}{N} \sum_{i=1}^N \vec{r}_i[t]$

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## Note

- The analysis is performed in a **d-dimensional** space
- Easy consequence: **convergence** for  **$n=2$**  robots using CoG algorithm

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## Analyzing CoG algorithm

**In fully synchronous model:** easy to analyze

**In semi-synchronous / asynchronous model (more involved):**

- Robots take measurements at different times, including while other robots are in movement
- Might cause oscillatory effects on the CoGs calculated by the various robots
- Might cause robots to pass each other by

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## Advantages of CoG algorithm

- Requires simple & efficient calculation - simple hardware, low computational effort.
- Applies to 1/2/3 dimensions and any # of robots.
- Bounded and simple-to-calculate rounding error
- Oblivious (i.e., requires no memory of previous actions & positions), hence
  - Memory-efficient
  - Self-stabilizing (i.e., finite number of transient errors cannot prevent eventual convergence).
- Prevents deadlocks (i.e., every robot can move at any given position unless already at center of gravity)

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## Results

- CoG algorithm is correct in both the **semi-synchronous** and **asynchronous** model, for any # of robots

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## Notation

Assume: Robots reside on  $x$ -axis.  
(Handle each axis separately)

$r_i[t]$  = location of robot  $i$  at time  $t$

$c_i[t]$  = CoG last calculated by robot  $i$  prior to time  $t$

$c[t]$  = True CoG at time  $t$

$H[t]$  = convex hull (smallest containing interval) of all  $r_i[t]$  &  $c_i[t]$

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## Monotonicity of convex hull

**Lemma:**  $H[t]$  cannot increase in time.

**Lemma:** In ASYNC, in any execution of CoG algorithm, over any  $O(n^2)$  time interval, the convex hull of robot locations + CoGs is **halved** (in each dimension separately).

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## Semi-synchronous model

**Lemma:**

In any execution of CoG algorithm, over any  $O(n)$  time interval, the convex hull of robots + CoGs is **halved** (in each dimension separately).

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## Fully synchronous model

**Lemma:** In any execution of CoG algorithm, robots achieve gathering in  $4 h[0] d^{3/2}$  time.

$d$ : #dimensions  
 $h[0]$ : max width of  $H[0]$

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## Problem 1.b: Self-stabilizing Deterministic Gathering

Petit et al., ALGOSENSORS 2009

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## We know that....

All solutions so far assume that the robots start from **distinct** positions

That is, even if the robots are oblivious, the solutions are **not truly self-stabilizing**

**Petit et al.** presented a **deterministic protocol** for solving the **self-stabilizing gathering**

■SSYNC

■Multiplicity Detection

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## We know that....



We know that the problem is unsolvable for 2 robots in SSYNC; hence, it is also unsolvable for an even number of robots

- The robots occupy at the beginning only two distinct positions on the plane, with  $n/2$  robots on each position

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## We know that....

**Petit et al** presented a **deterministic protocol** for solving the (self-stabilizing) gathering for an **odd** number of robots, starting from **any** configuration

■SSYNC

■Strong Multiplicity Detection

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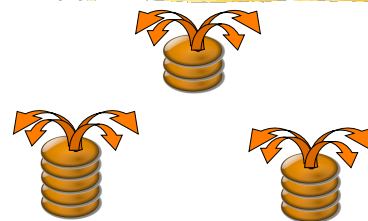
## Problem 2: Scattering

Petit et. al., FUN 2007

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## Scatter Problem

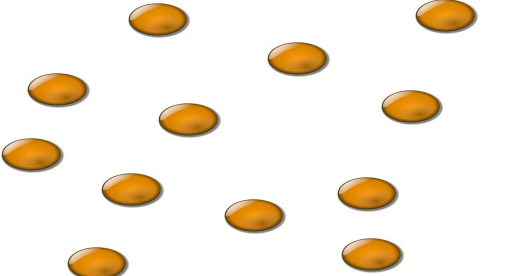


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## Scatter Problem



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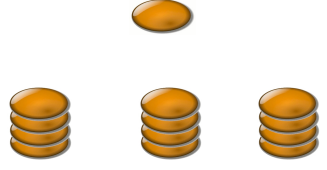
## Scatter Problem

- Studied in SSYNC
  - Convergence**  
Regardless of the initial positions of the robots, no two robots are eventually located at the same position
  - Closure**  
Starting from a configuration where non two robots are located at the same position, no two robots are located at the same position thereafter

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## Deterministic Scatter

~~Impossible~~



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## Randomized Scatter

No mult. Det.  No agreement 

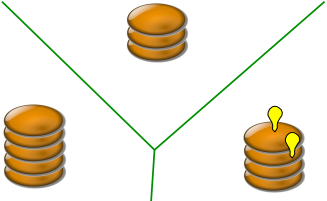
Compute the Voronoi Diagram  
If  $\text{Random}() = 0$   
then move arbitrarily in my cell

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## Randomized Scatter



No mult. Det.  No agreement 

— Compute the Voronoi Diagram  
If  $\text{Random}() = 0$   
then move arbitrarily in my cell

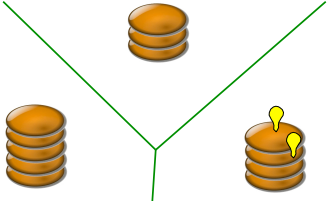


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## Randomized Scatter

— Compute the Voronoi Diagram  
— If  $\text{Random}() = 0$   
— then move arbitrarily in my cell



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### Randomized Scatter

Compute the Voronoi Diagram  
 If  $\text{Random}() = 0$   
 then move arbitrarily in my cell

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Compute the Voronoi Diagram  
 If  $\text{Random}() = 0$   
 then move arbitrarily in my cell

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### Randomized Scatter

- It could be used as a starting procedure for using previous gathering or arbitrary pattern formation
- Multiplicity detection is necessary to switch
- Eliminates the initial condition of having "distinct position"
- Randomized Self-Stabilizing Solutions

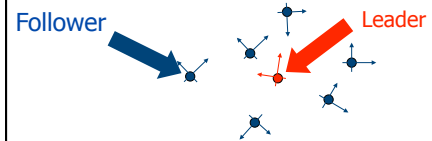
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## Problem 3: Flocking

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## Flocking Unlimited Visibility

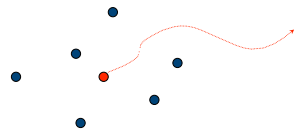


Followers recognize Leader  
No Agreement on Local Axes

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## Flocking Unlimited Visibility

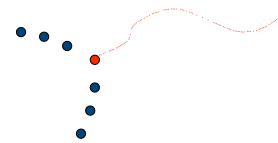


Leader acts independently (e.g., human driven)

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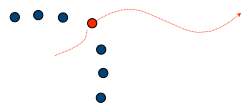
## Flocking Unlimited Visibility



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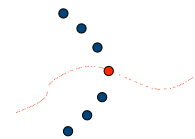
## Flocking Unlimited Visibility



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## Flocking Unlimited Visibility

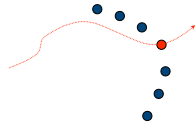


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## Flocking

Unlimited Visibility



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## Assumption

Time spent in *Look* and *Compute* is negligible  
w.r.t. the time spent in *Move*

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## The constraints

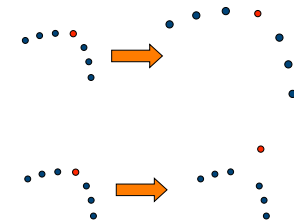
- **No** shared coordinate system
- **No** way to observe direction of movement
- **No** common velocity
- **Common** pattern
  - In order for the problem to be significant!

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## Necessary conditions

- Common distance unit
  - Otherwise, the followers could scale the pattern instead of following the leader
- Fast enough
  - The leader cannot be faster than the followers



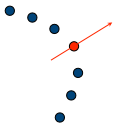
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## The Solution (Dam 2004)

**Followers** do not know the path to follow in advance

The algorithm lets **Followers** only form patterns that are symmetric w.r.t. direction of movement of the **Leader**



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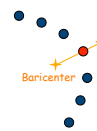
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## The Solution (Dam 2004)

**Followers** need a way to approximate the direction of movement of the **Leader**

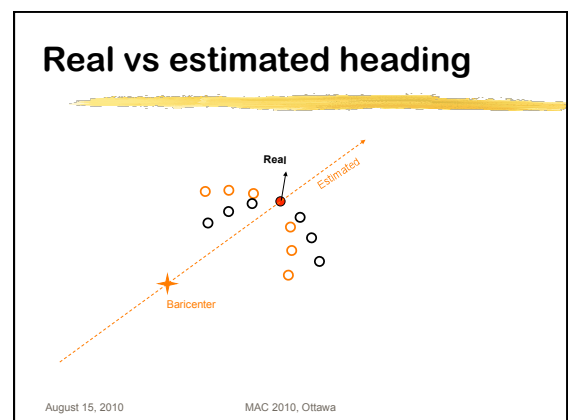
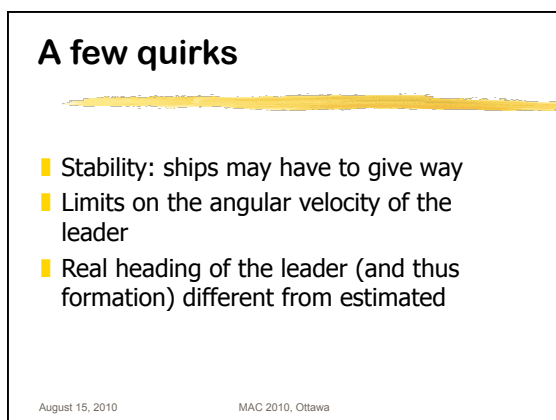
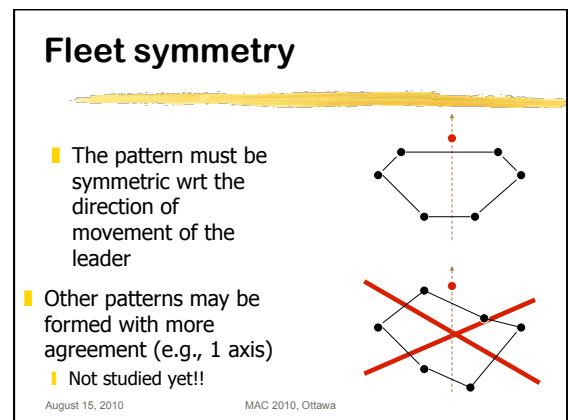
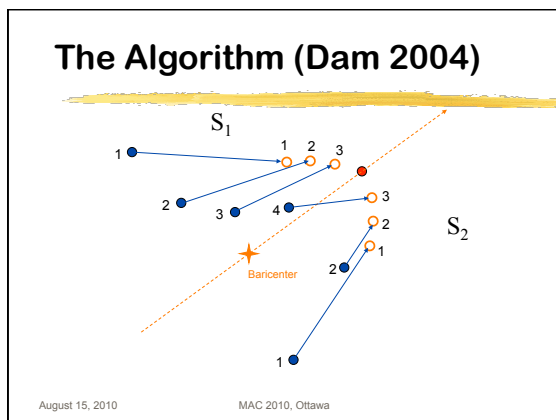
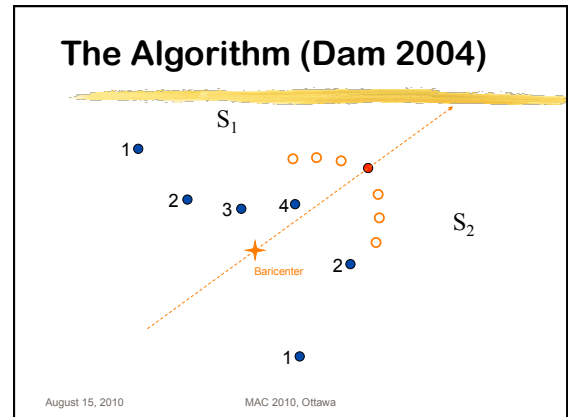
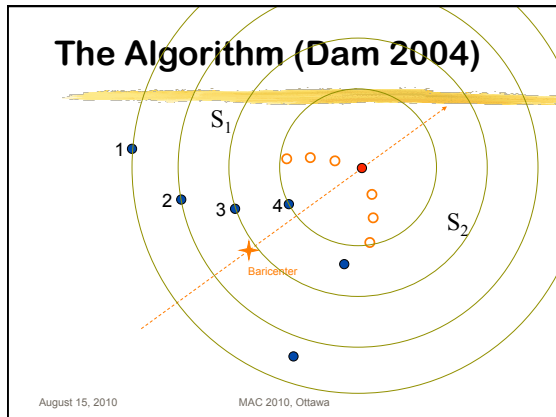
### Estimated direction of movement:

Given by axis passing through baricenter of **Followers'** positions and **Leader's** position



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## Measures

- Given a set of ships  $F$  and a set of points  $Z$  (the pattern), we define:

$$\Delta_{F,Z}(t) = \min_{\pi \in \Pi} \sum_{i=1}^{|F|} d(f_i(t), z_{\pi_i}(t))$$

- Experimental measures:

- Distance from estimated formation ( $\Delta_e$ )
- Distance from real formation ( $\Delta_r$ )

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## Experimental results

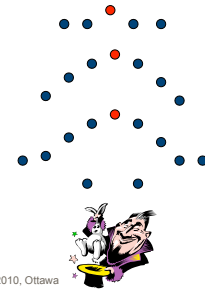
Tests of the solution with four formations:

Line (4)

Wedge (6)

Spread (10)

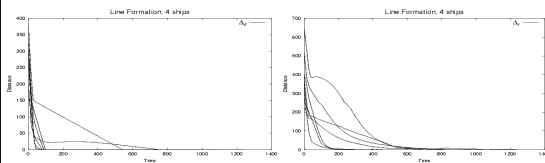
Random (2-8)



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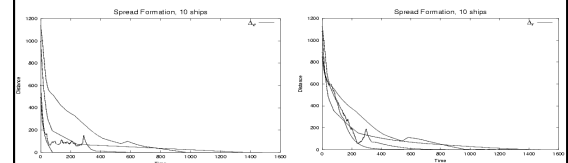
## Experimental results



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## Experimental results



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## Experimental results



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## Problem 4: Intruder

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## The intruder problem

- An **intruder** is trying to sneak through a restricted area
- A number of autonomous units (robotic **cops**) is patrolling the area
- The cops have to **surround** the intruder



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## The intruder problem

- Example: sensible infrastructures
  - Airfield runway
  - Logistic compound
  - Dam or power plant
- Example: hostile area
  - Battlefield
  - Police operation

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## The cops

- Units must be truly **autonomous**
  - Completely asynchronous
  - Undistinguishable (i.e., no IDs)
  - No explicit communications (i.e., no radio)
  - No shared knowledge
    - No common compass
    - No common chirality

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## The cops

- Units must be truly **autonomous**
  - Completely asynchronous
  - Undistinguishable (i.e., no IDs)
  - No explicit communications (i.e., no radio)
  - No shared knowledge
    - No common compass
    - No common chirality
- Algorithm must work with a **variable number** of cops
  - Cops could be knocked-off by the intruder

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## Computational model

- We consider an infinite plane
- Both the intruder and the cops are modeled as units that can freely move on the plane
  - All cops move according to a given **algorithm** (the same for all the units)
  - The intruder moves **independently** from other units; its course is not known in advance

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## Computational model

- Cops have **sensors** that report the **positions** of the intruder and of other cops
- Cops are **oblivious** – they have no memory, and do not rely on stored information about the past

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## Computational model

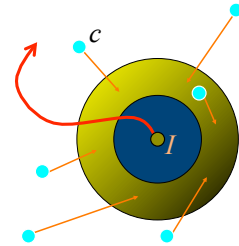
- As in the flocking, we assume that
  - Time spent in *Look* and *Compute* is negligible w.r.t. the time spent in *Move*

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## Formalizing the problem

- The cops must occupy a **ring** around the intruder:
 
$$r_1 \leq \text{dist}(c, I) \leq r_2$$
- The cops must be evenly spaced, minimizing the maximal "escape hole" in the ring

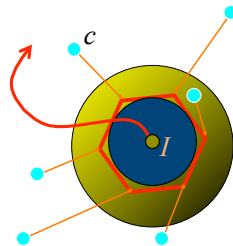


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## Formalizing the problem

- Both conditions can be met by forming a regular  $n$ -gon of characteristic angle  $2\pi/n$  and radius  $r_1$



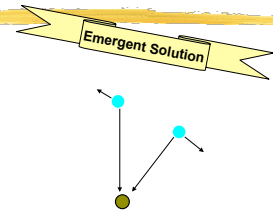
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## Two Algorithms: Heuristic (FUN 2004)

### Heuristic approach

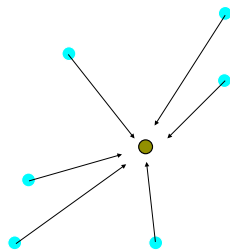
- All robots are subject to two forces:
  - Attractive, towards the enemy
  - Repulsive, mutual
- Parameters are tuned so that the equilibrium is a solution for the problem



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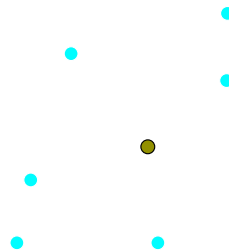
## The HEUR-S Algorithm



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## The HEUR-S Algorithm



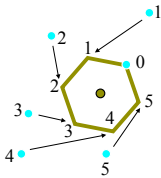
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## Two Algorithms: LAT

Precise Solution



### Algorithmic approach

- The robots reach an agreement on their ordering
- Each one establishes its own target
- They reach the target (taking care not to overtake each other)

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## The LAT algorithm (idea)

- The closest cop to the intruder is identified and declared *Chief*

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## The LAT algorithm (idea)

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## The LAT algorithm (idea)

August 15, 2010

MAC 2010, Ottawa

## The LAT algorithm (idea)

- The closest cop to the intruder is identified and declared *Chief*
- The Chief moves towards the intruder, up to a distance  $r_1$

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## The LAT algorithm (idea)

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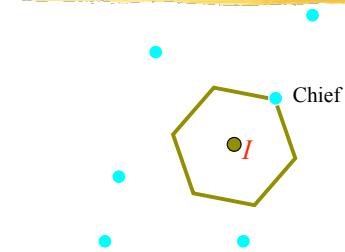
### The LAT algorithm (idea)

- The closest cop to the intruder is identified and declared *Chief*
- The Chief moves towards the intruder, up to a distance  $r_1$
- The other cops scale and align the  $n$ -gon so that the center is on  $I$  and a vertex on the Chief

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### The LAT algorithm (idea)



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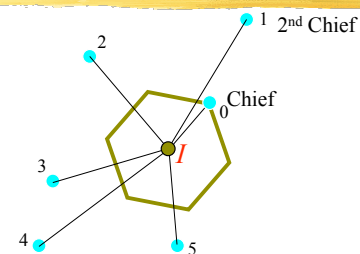
### The LAT algorithm (idea)

- The closest cop to the intruder is identified and declared *Chief*
- The Chief moves towards the intruder, up to a distance  $r_1$
- The other cops scale and align the  $n$ -gon so that the center is on  $I$  and a vertex on the Chief
- All cops are lexicographically sorted according to their angle with  $I$ -Chief
- Orientation is Chief  $\rightarrow$  2<sup>nd</sup> Chief

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### The LAT algorithm (idea)



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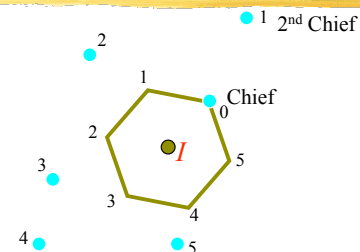
### The LAT algorithm (idea)

- The closest cop to the intruder is identified and declared *Chief*
- The Chief moves towards the intruder, up to a distance  $r_1$
- The other cops scale and align the  $n$ -gon so that the center is on  $I$  and a vertex on the Chief
- All cops are lexicographically sorted according to their angle with  $I$ -Chief
- Orientation is Chief  $\rightarrow$  2<sup>nd</sup> Chief
- The  $i$ -th cop in the ordering moves (up to  $\varepsilon$ ) toward the  $i$ -th vertex of the scaled and aligned  $n$ -gon

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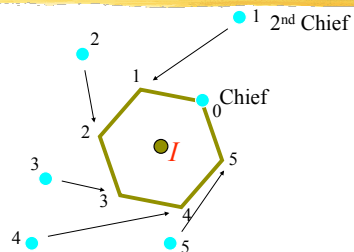
### The LAT algorithm (idea)



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## The LAT algorithm (idea)



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## Some pitfalls....

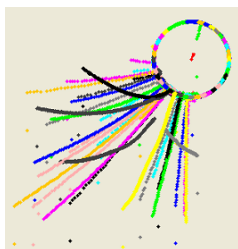
- Multiple candidate chiefs
  - All cops wait for the intruder to move, breaking the tie
- A cop may have to go to the opposite side of the ring
  - Perform a *sideway step* to avoid a change of Chief

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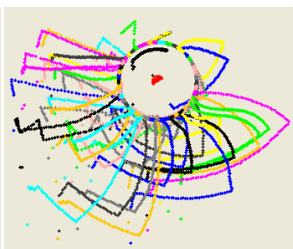
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## Two Algorithms

HEUR-S trace



LAT trace

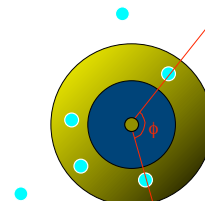


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## Evaluation of the algorithm

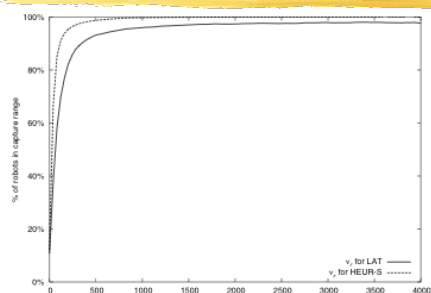
- Numerical simulations
- Two kind of measures:
  - What fraction of the cops is inside the target ring?
  - How large is the largest hole in the ring, w.r.t. the optimal hole?



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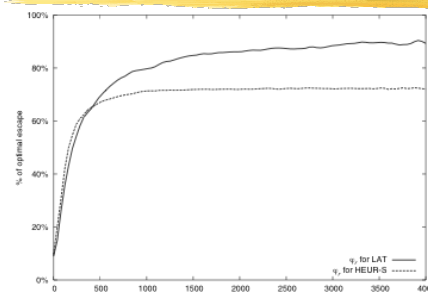
## Fraction of Robots in the Capture Area



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## Size of Largest Hole (relative)



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## Adaptiveness

- The algorithm adapts to dynamic changes in the number of cops
  - Cops can join
  - Cops can leave

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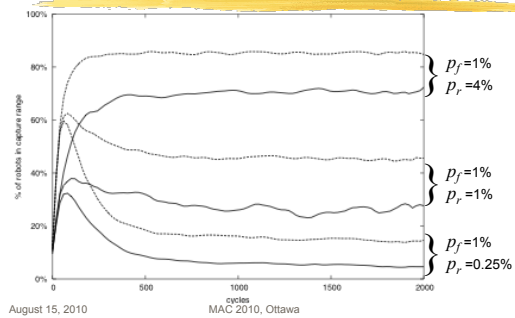
## Fault Tolerance

- Robots could be blocked, knocked off by the enemy, or otherwise suffer **faults**
- We consider **transient** faults
  - A faulty robot stops moving, but continues sensing and computing
  - Fault model based on two parameters:
    - $p_f$  – probability of occurrence of a fault
    - $p_r$  – probability of resuming normal behaviour
- Faults occur indefinitely

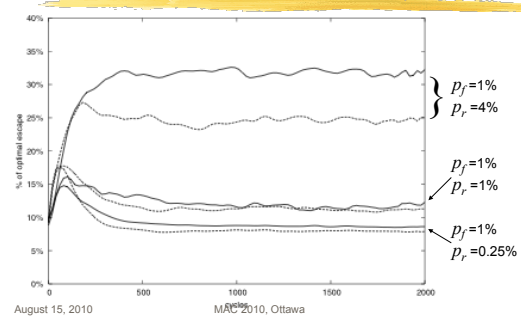
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## Robots in the Capture Area (in the presence of faults)



## Size of the Largest Hole (in the presence of faults)



## Summary

- Gathering
  - ASYNC
  - CoG (ASYNC and SSYNC)
  - Self-Stabilizing Gathering in SSYNC
- Scattering (SSYNC)
- Flocking
- Intruder

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Switzerland

# THE END

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