Gathering Problem Assorted

Taisuke Izumi
(Nagoya Institute of Technology)

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Gathering with Local-Multiplicity Detection

Taisuke Izumi (Nagoya Institute of Technology)
Tomoko Izumi (Ritsumeikan University)
Sayaka Kamei (Hiroshima University)
Fukuhito Ooshita (Osaka University)

Gathering Problem

- All robots meet at one point
- Basic Results (Deterministic Algs.)
  - Impossible [Suzuki and Yamashita SICOMP'96]
    - ATOM (Semi-sync, SYm) & 1-bounded
    - #robots = 2
    - No multiplicity
    - No agreement of coordinate systems
    - Oblivious

Gathering Problem

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We use terminology "ATOM"

In the following slides of the first topic,
we assume them without explicit statement …

Employing Randomization

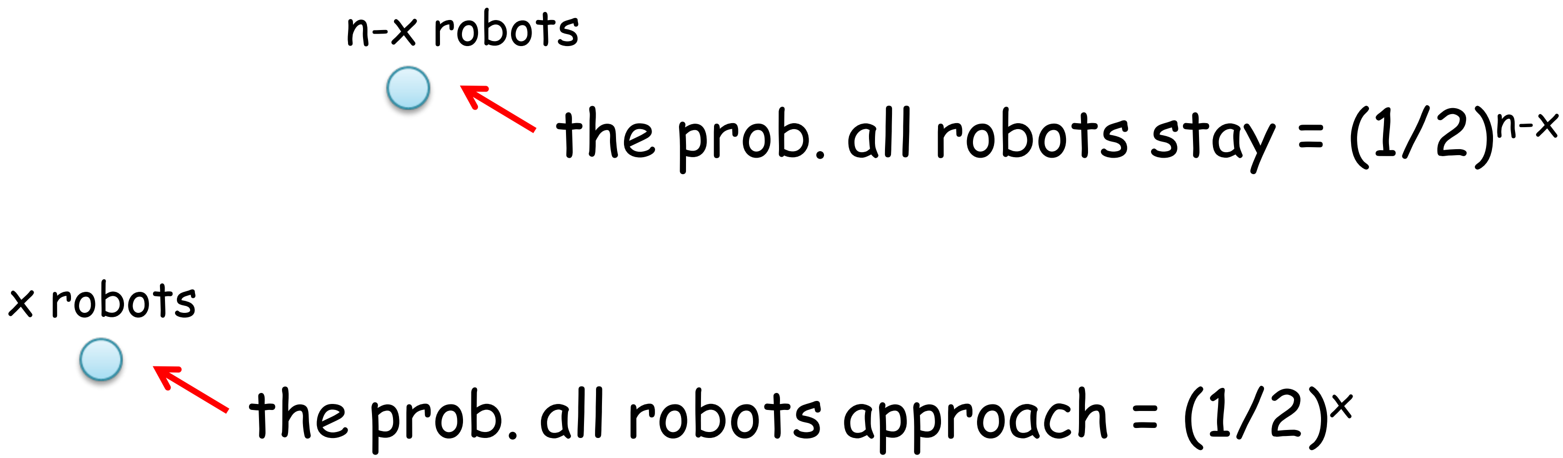
- Key difficulty : Hardness of symmetry breaking

→ Randomization will be helpful !!

- Randomized gathering is easy
  - A simple algorithm for two robots
    - stay with probability 1/2
    - approach to the other with probability 1/2

Exponential Growth of Running Time

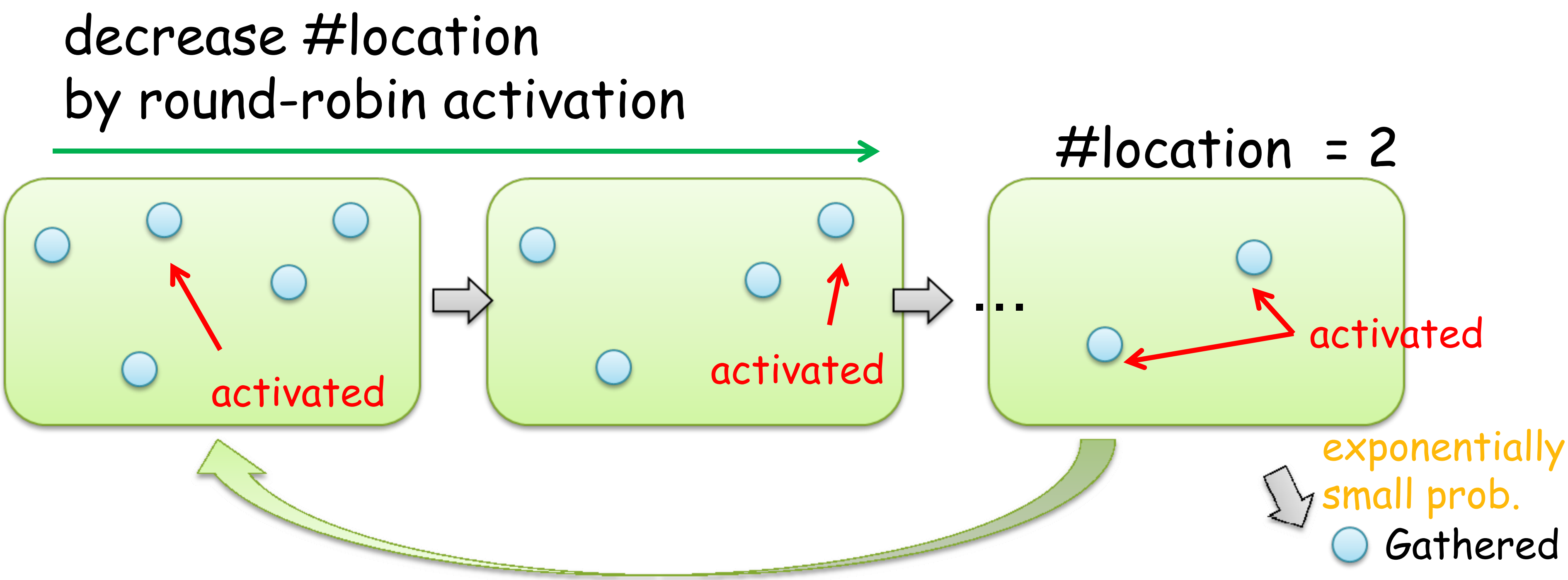
- It achieves gathering for n robots, but..
  - not poly(n) , but exp(n) expected running time



- Can we have poly(n) algorithm?

Impossibility

- No probabilistic gathering with poly(n) expected running time
  - ATOM & 2-bounded
  - No multiplicity



The Question

- The impossibility implies an additional assumption is necessary to achieve poly(n)-time probabilistic gathering

What is the weakest assumption?

Our Focus : Multiplicity Detection

- Detection capability of two or more robots on the same location

- Known class of multiplicity detection
  1. No multiplicity : observed as a single robot
  2. Weak multiplicity : detect more than one
     1. the observer cannot know the exact #robots
  3. Strong multiplicity : detect #robots

There exists a deterministic gathering alg.
CORDA & ∞-bounded, no initial multiple location
[Cieliebak et al. TCS'05]

The Cases of **Initial Multiple Locations**

- Deterministic algs. : impossible
  - The argument of two points with n/2 robots
  - ATOM & 1-bounded, Strong
- Randomized algs. : possible[Clement et al. IPL'10]
  - ATOM & ∞-bounded,
    predictable and unlimited move
- Complexity
  - Strong : O(n) movements, O(1) async. rounds
  - Weak : O(n ・logn ~~loglogn~~) movements,
    O(logn~~loglogn~~) asyhc. rounds

Locality of Multiplicity Detection

- How about further weaker capability?
  - New criterion of multiplicity ：Locality
    - The observer can detect only the multiplicity of **its current location**
- New five classes of multiplicity detection
  - No multiplicity
  - Local weak multiplicity
  - Global weak multiplicity
  - Local strong multiplicity
  - Global strong multiplicity

Known Randomized Solutions

- Local Strong
  - Upper bound: $O(n)$ move, $O(1)$ async. rounds
    - ATOM & 1-bounded, initial multiple locations, predictable and unlimited move
    - but easy to extend it to k-bounded for $k < \infty$
- Local Weak
  - Lower bound: $\Omega(\exp(n))$
    - ATOM & 1-bounded, initial multiple locations
  - Upper bound: $O(n)$ move, $O(1)$ async. rounds
    - ATOM & 1-bounded, no initial multiple locations, predictable and unlimited move

Known **Randomized** Solutions

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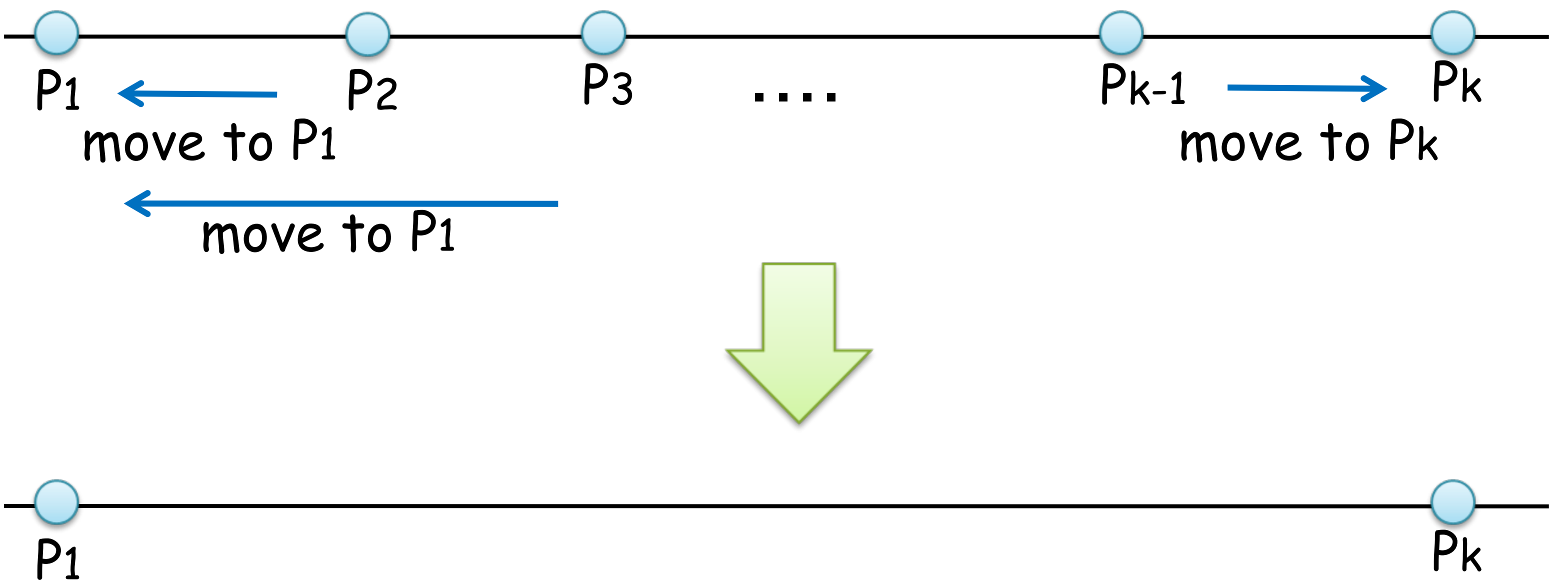
Algorithm for Local-Strong Multiplicity

- Composition of two subalgorithms
  - ML (Making Line)
    - All robots are lined with O(n) movements
    - no randomness necessary
    - no multiplicity detection necessary
  - GfL(Gathering from Line)
    - Probabilistic Gathering on one-dimensional space
    - Taking O(n) movements and One round, all robots are gathered with constant prob.
    - Multiplicity detection plays an important role

Algorithm GfL(1)

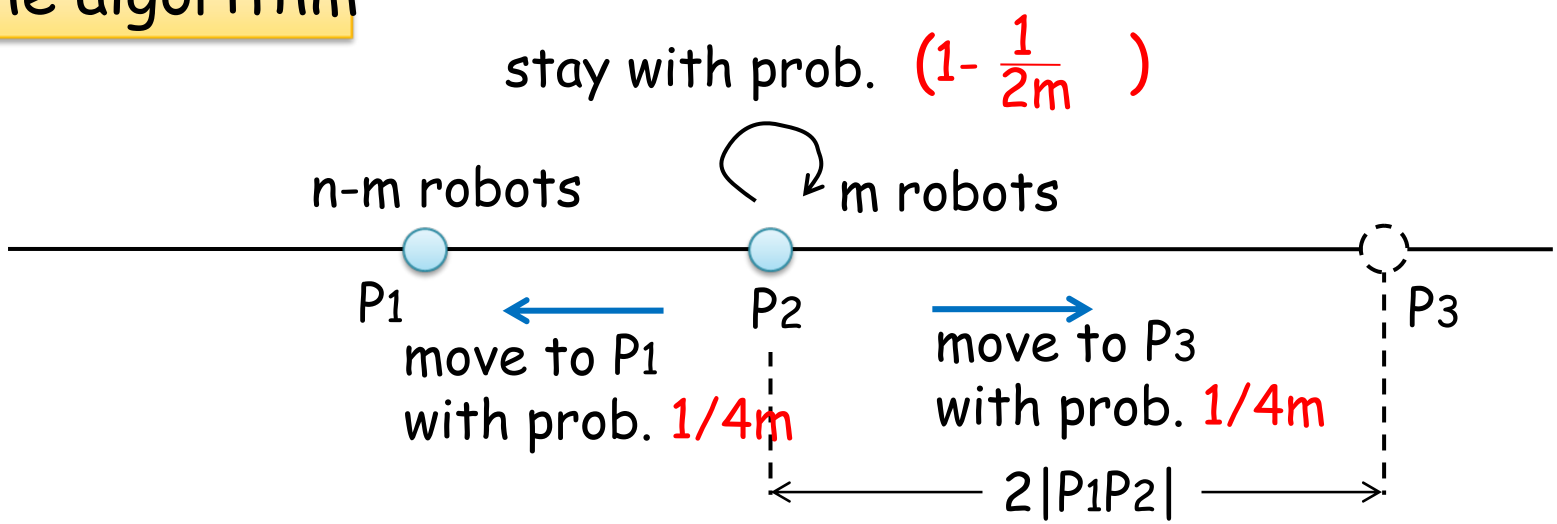
- First, reduce to a two-point configuration
  - Inner robots moves to the nearest endpoint



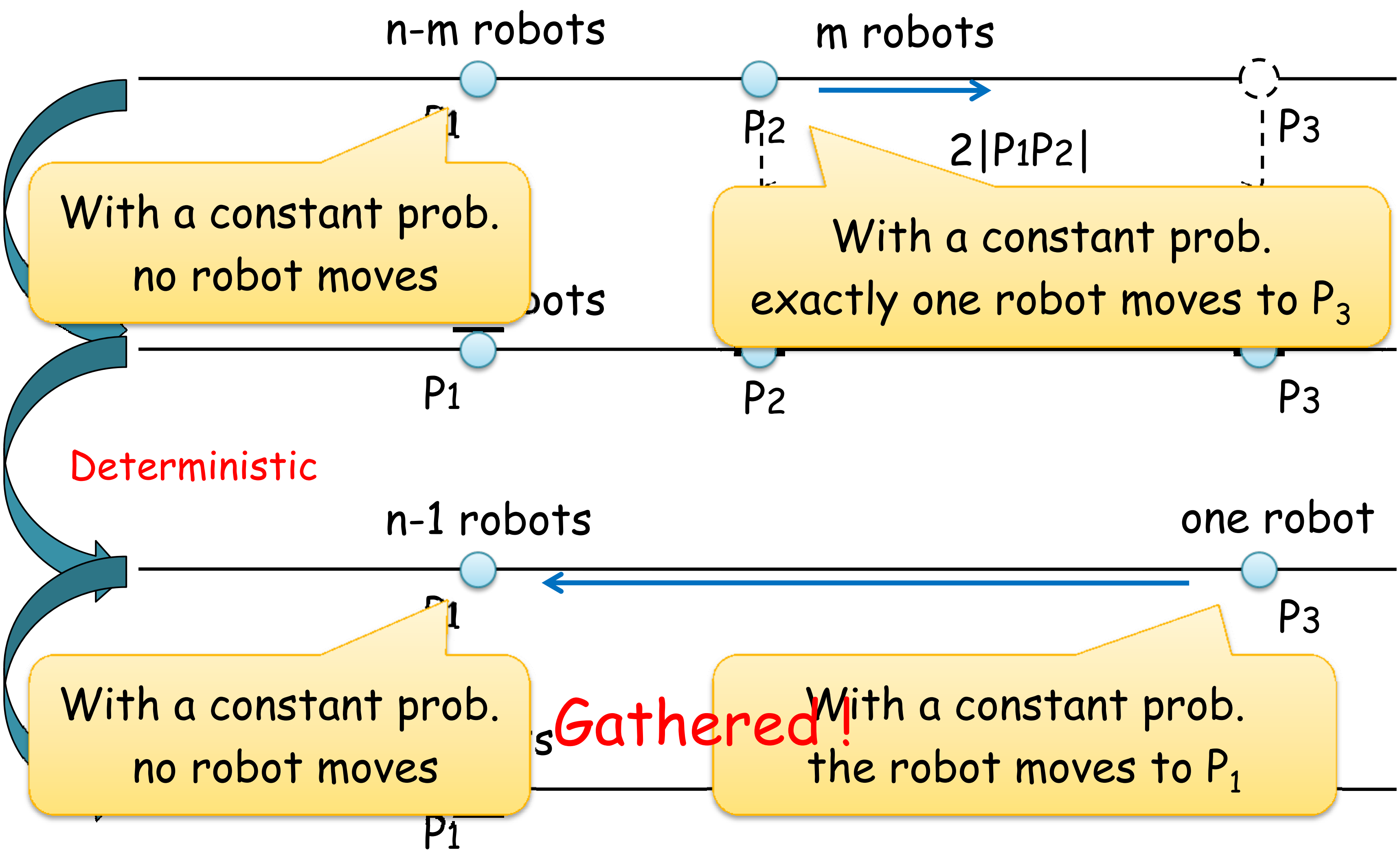
Algorithm GfL(2)

- Two points to one
  - Idea: Higher multiplicity stays with higher probability

The algorithm



Expected Behavior (Occurring With Const. Prob.)



Known **Randomized** Solutions

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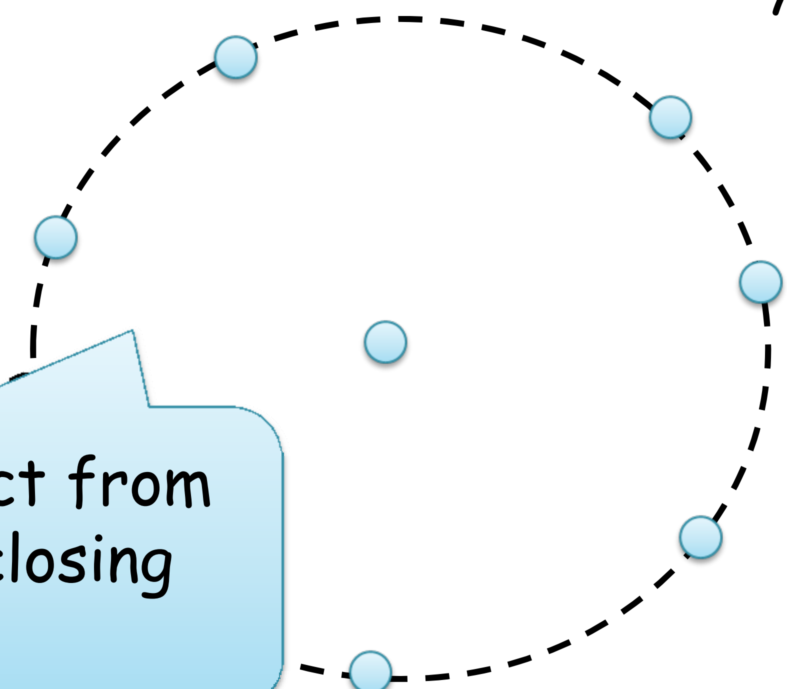
The difficulty

- How can we avoid two-point symmetric case?
  - Only the way is using multiplicity information
    - But #robots is not available
  - So, we need the following situation

- The problem caused by locality
  - No robot can detect which location is multiple

Algorithm for Local-Weak Multiplicity(1/3)

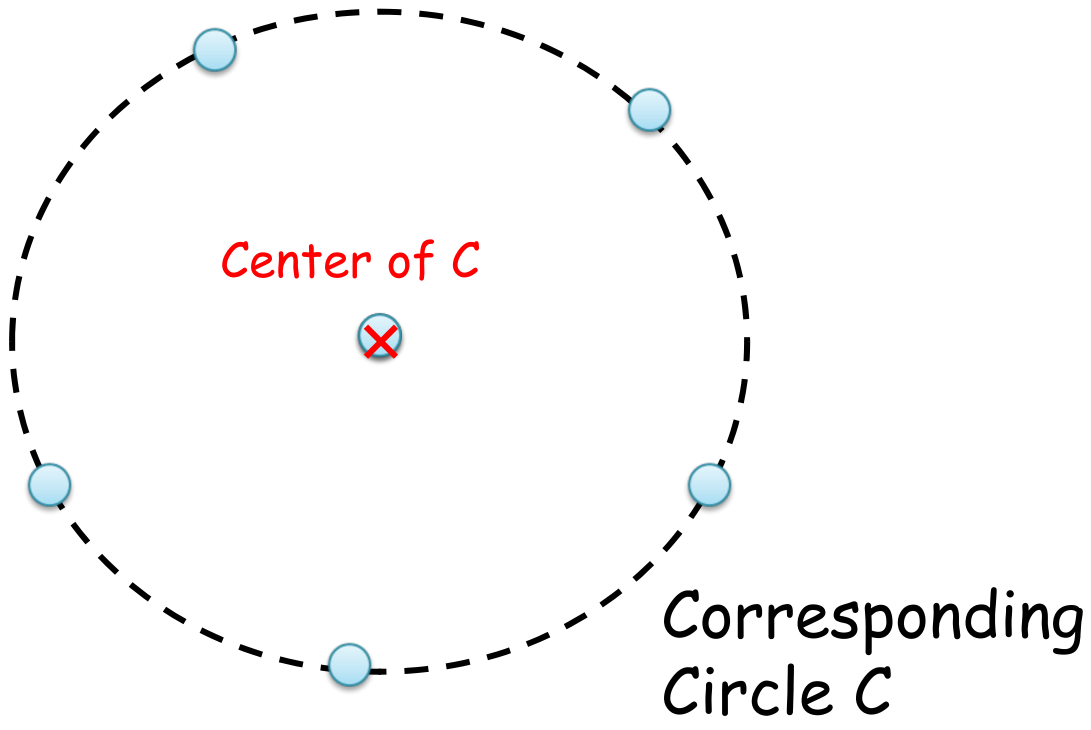
- Invariant : Circular Configuration
  - There exists a circle *C* (corresponding circle) s.t.
    - At least one robot is on the center of *C*
    - All other robots are on the boundary of *C*



Easy to construct from the smallest enclosing circle

Algorithm for Local-Weak Multiplicity(2/3)

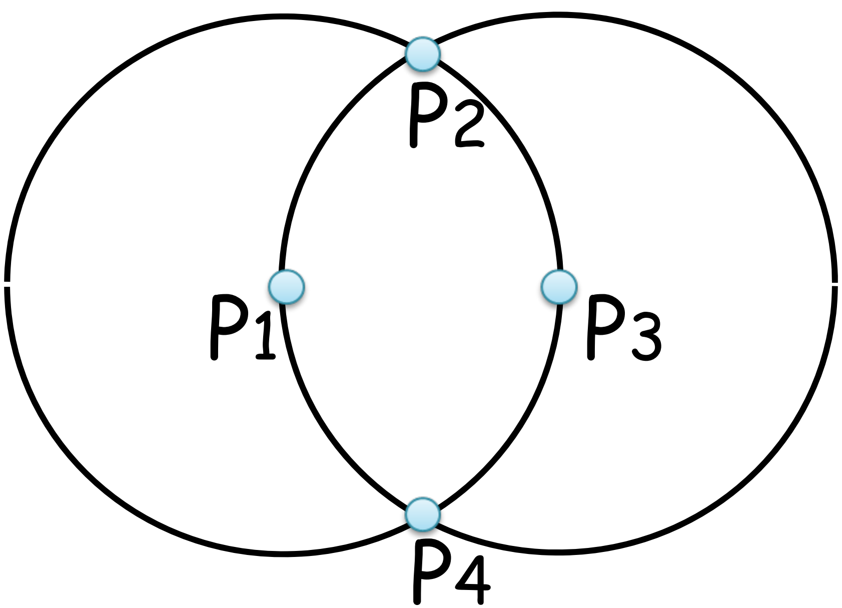
- All robots on the boundary go to the center of C
  - The Center is almost invariant until #location = 2

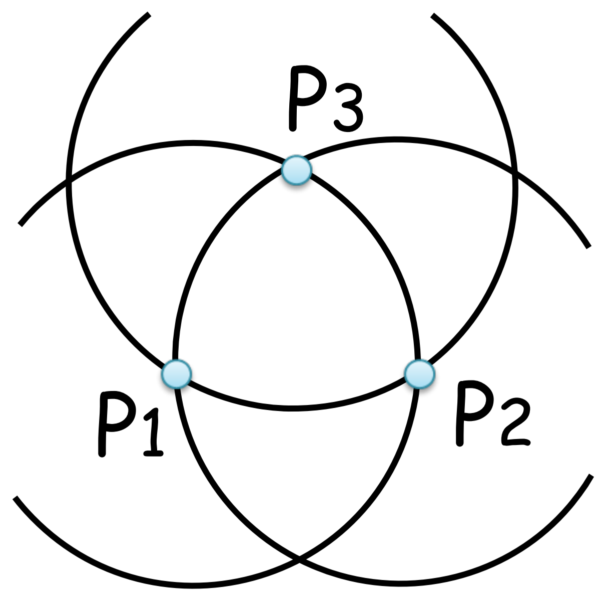


Algorithm for Local-Weak Multiplicity(2/3)

- Two exceptional cases
  - Regular Diamond
  - Regular Triangle

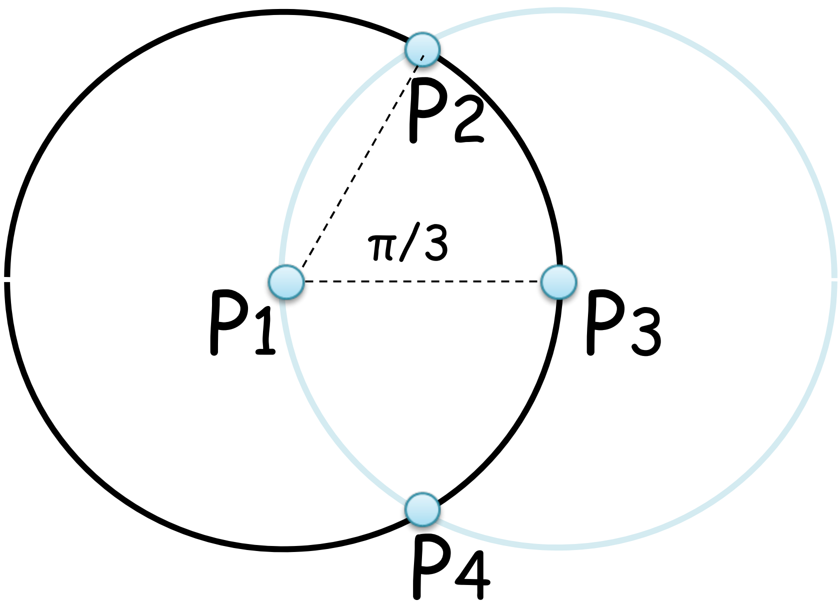
Circular but C is not uniquely determined

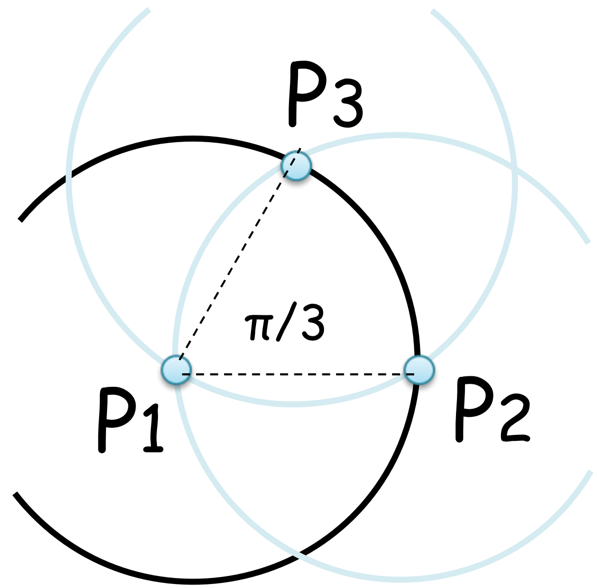




Algorithm for Local-Weak Multiplicity(2/3)

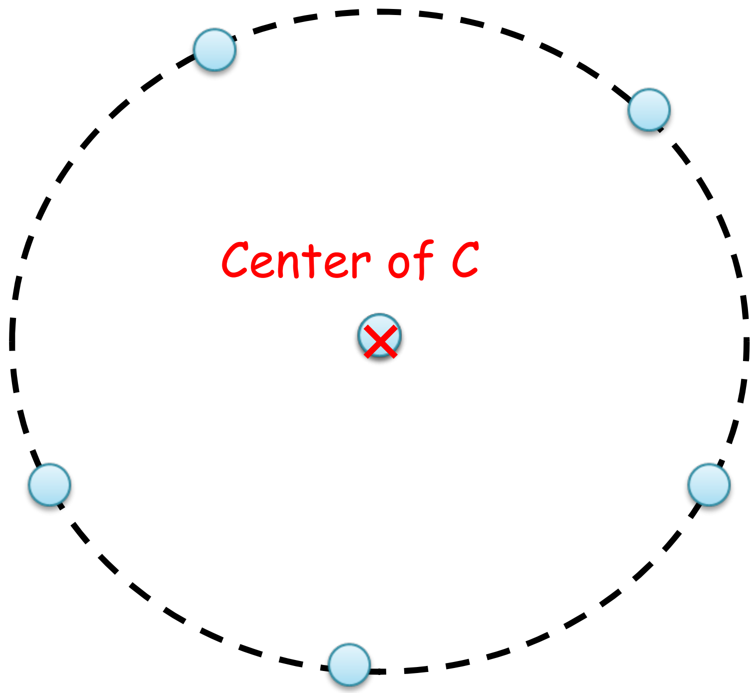
- The necessary condition to occur exceptional cases
  - Two robots on the boundary form the center angle π/3





Algorithm for Local-Weak Multiplicity(2/3)

- The main idea:
  - “Shake” center angles via randomization
  - Moves to the center after angle $\pi/3$ disappears



Open Problems (1)

- Deterministic alg. with Local multiplicity
  - Strong, ATOM & k-bounded,
    Predictable and unlimited move,
    No initial multiple points
- Randomized alg. with Local multiplicity
  - Strong/Weak, Atom & k-bounded,
    Unpredictable move,
    No initial multiple point
- How can we measure the complexity on unpredictable move models?
  - An idea: measuring on predictable models.

Open problem (2)

- Lower Bound for Global-Weak multiplicity
  - Known Upper bound: O(log n) (maybe)
  - Is it optimal?
- Randomized Gathering on CORDA
  - Initial multiple points

Strong Impossibility Results for Byzantine Gathering via BG-simulation

Taisuke Izumi[1]
Zohir Bouzid[2]
Sebastien Tixeuil[2]
Koichi Wada[1]

1 Nagoya Institute of Technology
2 *Université Pierre et Marie Curie - Paris 6*

Byzantine Gathering Problem

- All correct robots meet at one point
- Known Results
  - Impossible for n=3 and f=1
    [Agmon and Peleg SODA'03,SICOMP'06]
    - ATOM, ∞-bounded, uniform, oblivious, no agreed coord. , deterministic
  - Impossible for general n and f
    [Clement et al. DISC'06]
    - ATOM, n/f-bounded, … agreed coord. , determ…
  - Possible for general n and f s.t. n > 3f+1
    [Agmon and Peleg SODA'03,SICOMPY06]
    - Full-Sync, uniform, oblivious, no agreed coord., deterministic

Sorry, I made mistake yesterday

Faulty Robots

- Fault model
  - Crash : Stop working
  - Byzantine : Arbitrary Behavior
- We assume Byzantine behavior is bound by k-bounded scheduler constraint
  - If we remove this assumption, our impossibility is strengthened

Impossibility for Stronger Condition

- All of previous results derive from :

> Geometric Argument
> + Hardness of Symmetry Breaking

- Not easy (not possible?) to apply them to non-oblivious, non-uniform, or agreed-coordinate-system robots

Our Result

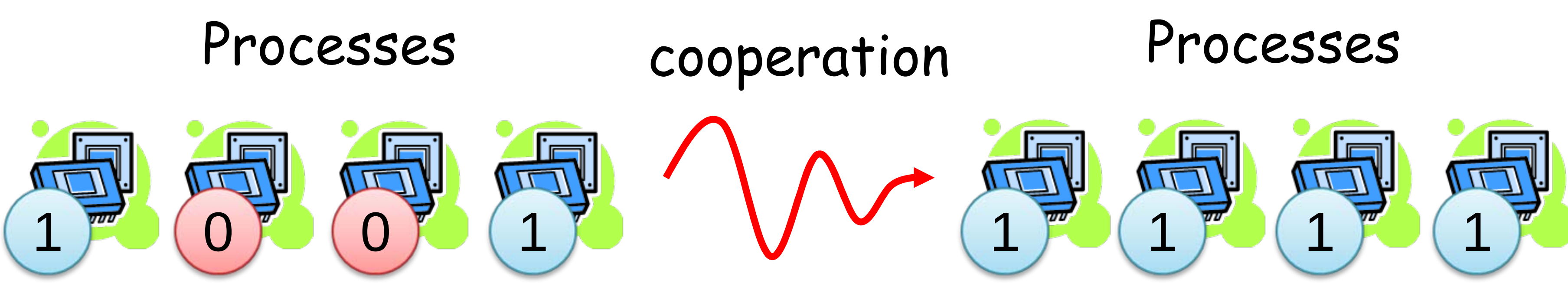
- Strong Impossibility for Byz. Formation (Any pattern)
  - general n and f
  - ATOM, (n/f+1)-bounded
  - non-oblivious
  - non-uniform
  - agreement of coordinate systems
    - both orientation and direction

New versatile proof technique not relying on geometric argument

The Proof Idea : An Analogy to Consensus Problem

(Byzantine) Gathering ≒ Consensus Problem

- (Binary) Consensus problem
  - **Not a problem on robot systems**
  - Each process first proposes one or zero
  - All correct processes decide a common value
  - The decision must be one of proposals



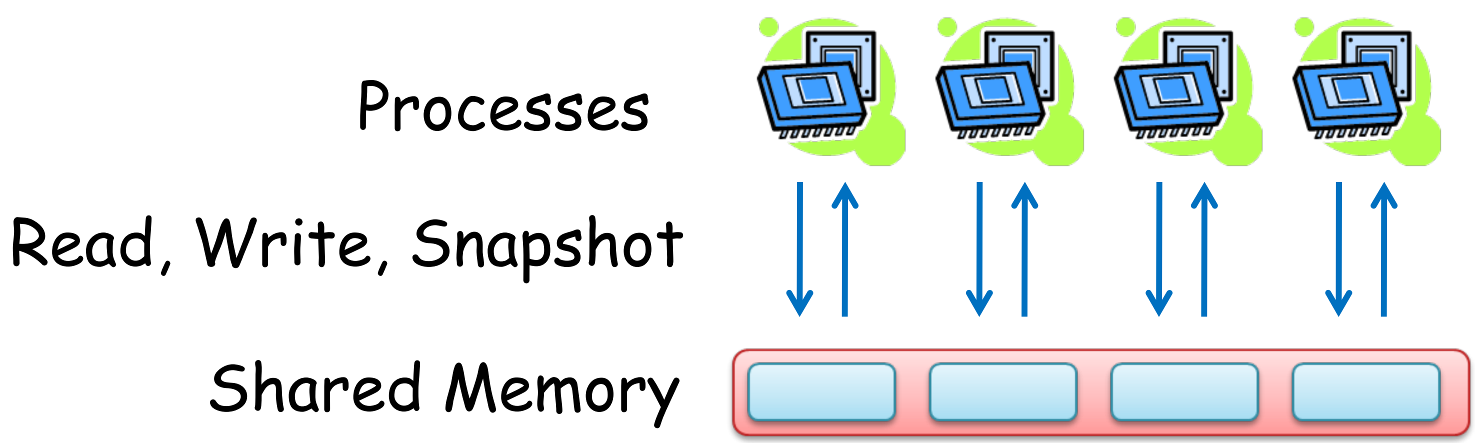
Reduction from Shared-memory Consensus

- Consensus problem is not solvable
  - Asynchronous shared-memory systems
  - One crash fault (not Byzantine!)
- Reduction Strategy (Case of f = 1)
  - Simulate 1-Byzantine Robot system on Asynchronous 1-Crash shared-memory system
  - Solving Consensus via Gathering

Asynchronous Atomic Snapshot Model

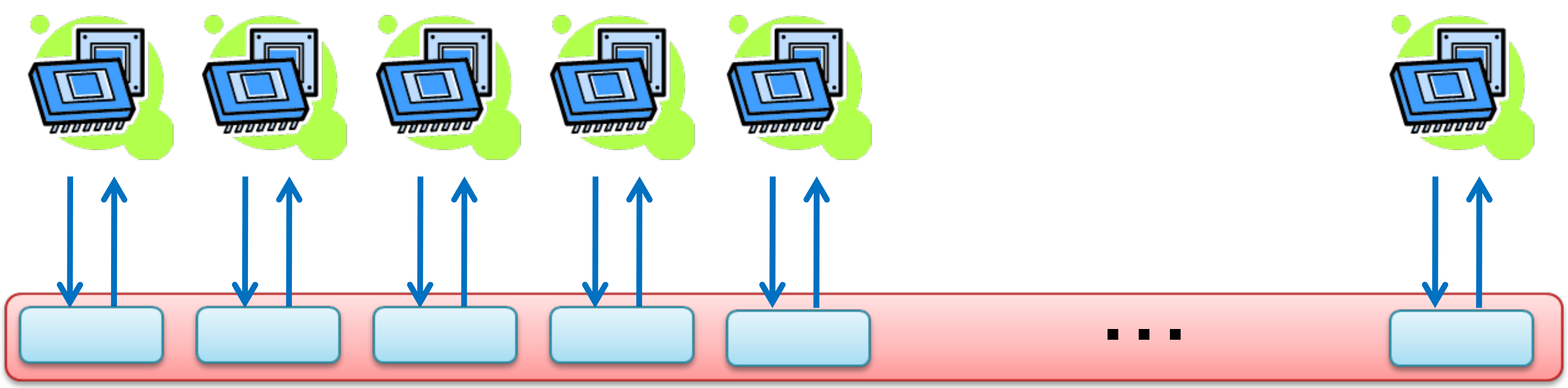
- We use a variation of Asynchronous shared-memory models
  - Read, Write, and Snapshot

    Atomic reading of all shared memory
  - Equivalent to the standard model
    - 1-crash resilient consensus is not solvable



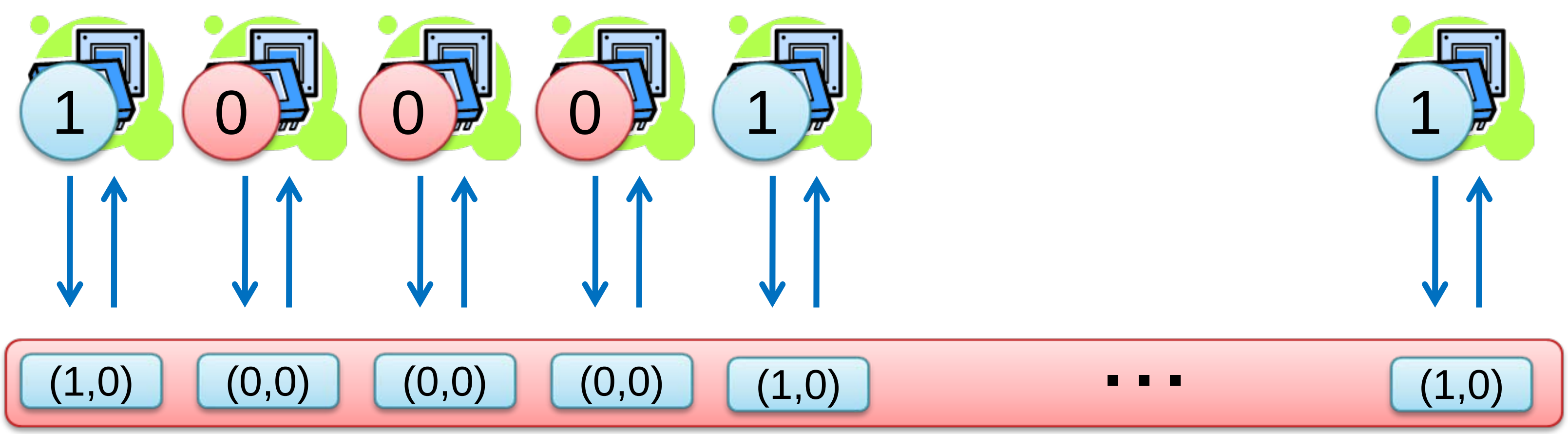
Naïve Idea(1/2)

- Prepare the shared array of size n
  - i-th entry = the state of i-th robot
    - "state" includes the current location and internal state of i-th robot
- The i-th process simulates i-th robot
  - Look = Atomic Snapshot
  - Move = Write to the i-th entry



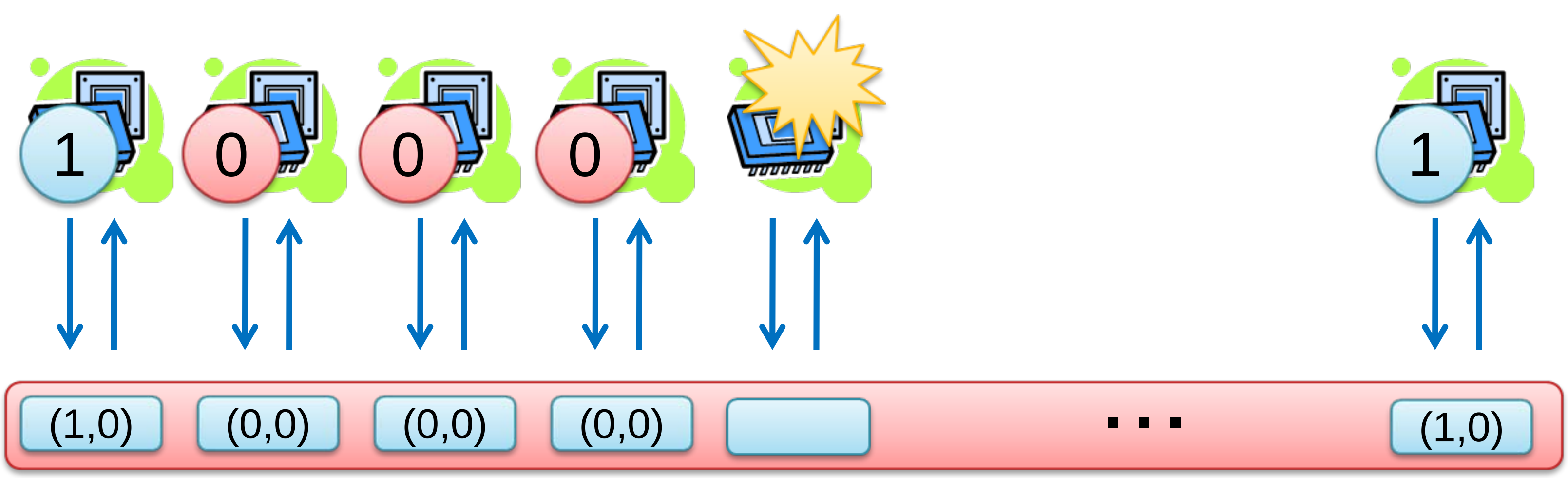
Naïve Idea(2/2)

- For each proposal v
  - Set initial location to (v, 0)
- Decision
  - All robots gathered at (1,0) → decide(1)
  - Otherwise → decide(0)



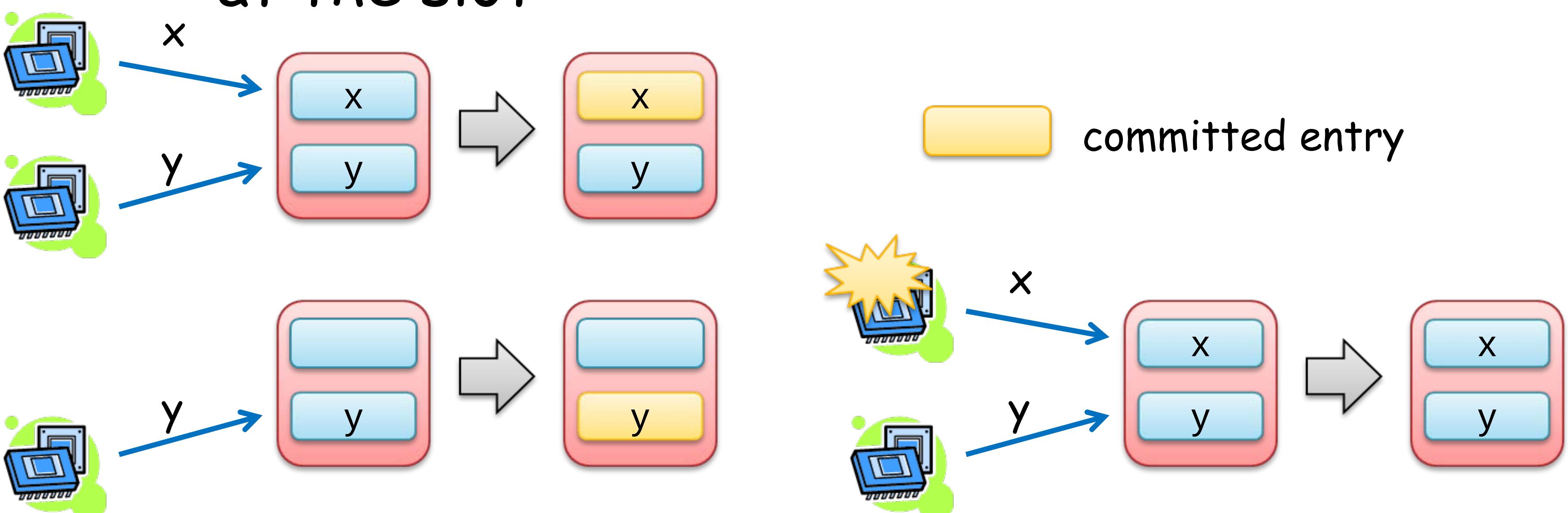
Naïve Idea does not work

- The simulation is not 1-crash resilient
  - If some process is initially crashed, one robot is lost → Simulation failed!



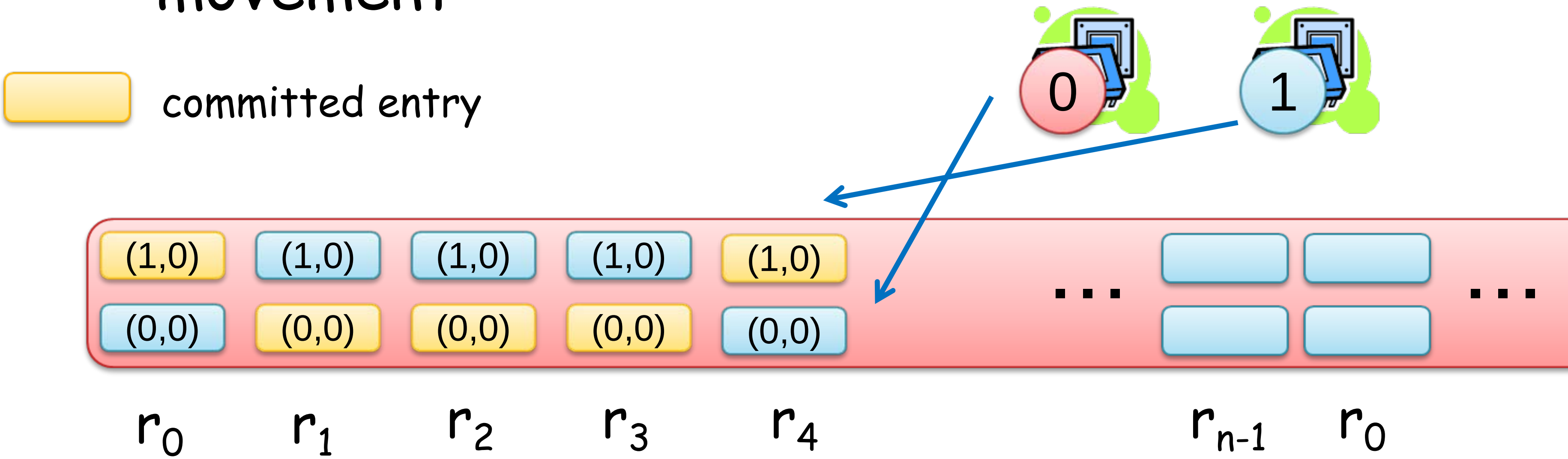
Our 1-reisilient Simulation

- Key Technology: BG-simulation
  [Borowski and Gafni, STOC'93]
  - Simulation by two processes
- Use Slot structure
  - One value is committed when no process is at the slot



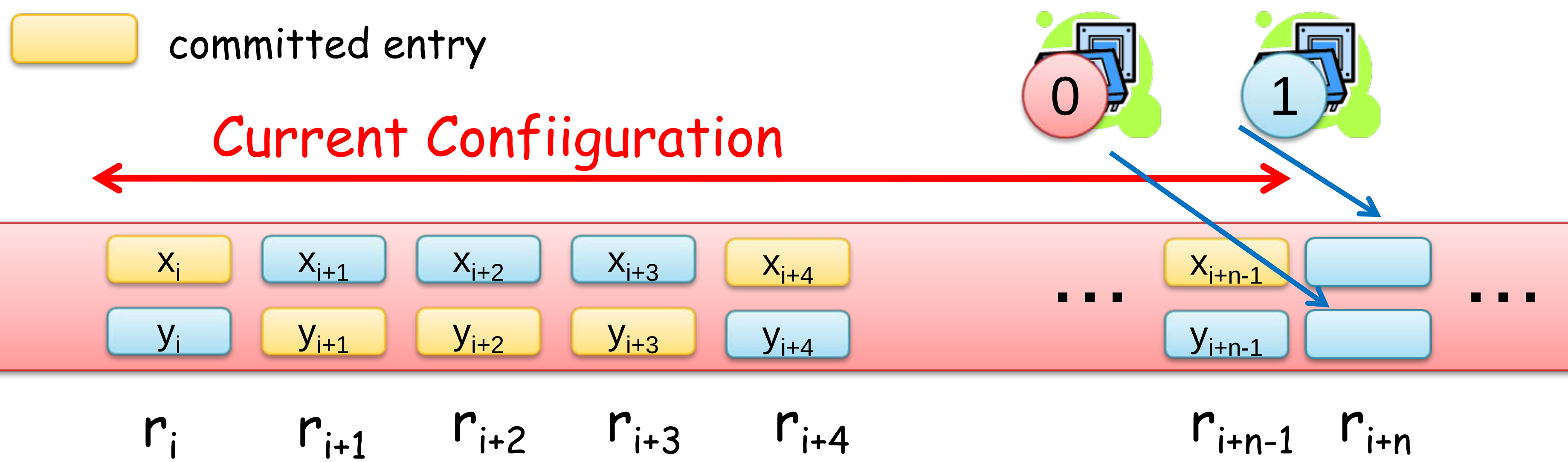
Our 1-resilient simulation

- Array of slots with infinite size
  - One slot = one movement of robot
- Each process simulates all n robots in round-robin manner
  - k-th Slot = (k mod n+f)-th robot's movement



Our 1-resilient simulation

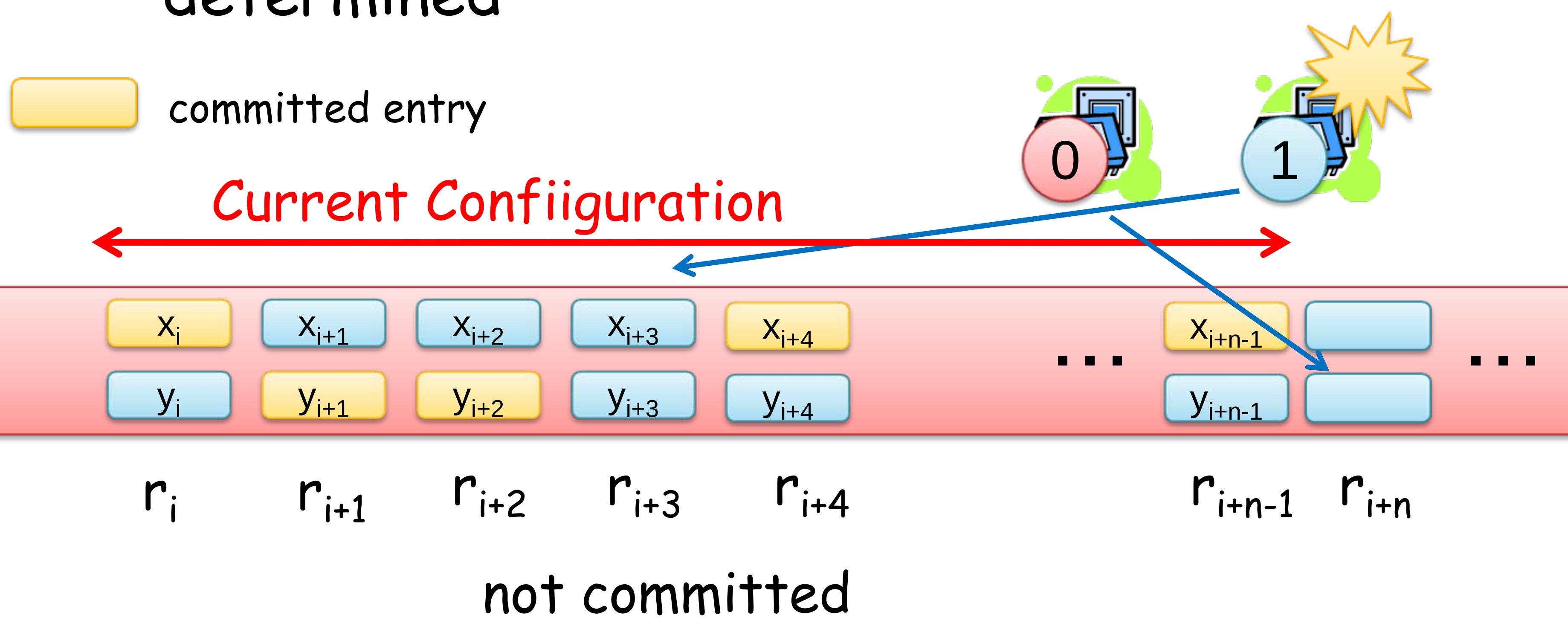
- Simulation of k-th Slot
  - Observe (k-n)-th to (k-1)-th
  - Compute the destination and write it to k-th



Our 1-resilient simulation

- Process crash (or slowdown)
  - One slot may be uncommitted

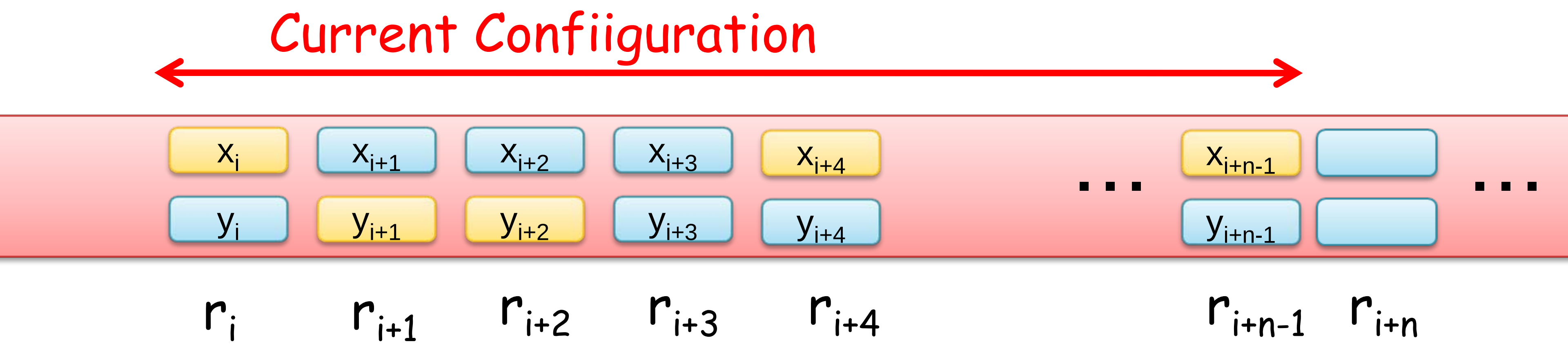
  → Current configuration cannot be determined



Our 1-resilient simulation

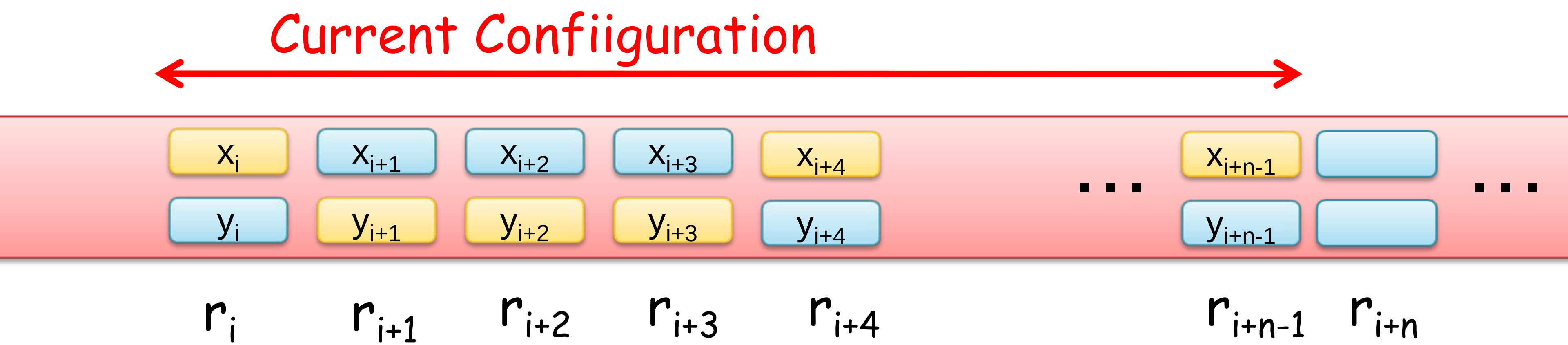
- Uncommitted slot has two values
  - only one is committed
- Main Idea: uncommitted value = Byzantine
  - Actually, we simulate (n+1)-robot systems!
  - Interpretation of the below situation
    - Current Conf. = (x_i, y_{i+1}, y_{i+2}, x_{i+3}, x_{i+4} … x_{i+n-1}, $\underline{y_{i+3}}$)

Byzantine



Our 1-resilient simulation

- When all slots are correctly committed
  - Add a "dummy" location for Byzantine robots
  - Interpretation of the below situation
    - Current Conf. = $(x_i, y_{i+1}, y_{i+2}, x_{i+3}, x_{i+4} \ldots x_{i+n-1}, (0,0))$



Conclusion

- Strong Impossibility Result for Byzantine Formation
- New Proof Technique
  - Reduction from the consensus problem on Asynchronous Atomic Snapshot models
  - Reduction = simulation algorithm
    - A number of tricks to achieve 1-resiliency
- Classical DC theory helps robot theory!

Open problems

- Deterministic Byzantine gathering
  - ATOM & k-bounded for k < n/f, non-oblivious, non-uniform, agreed coord. , predictable and unlimited move
- poly(n)-Randomized Byzantine gathering
  - Weaker condition than the above
- Further strong impossibility for CORDA
  - Conjecture: Only one-shot Byzantine behavior prevents the gathering

Open Problems

- Byzantine Formation
  - Some patterns are probably impossible
    - Even if we assume strong assumptions
  - What are formable patterns?
- Finding other bridges to classical DC theory
  - e.g. reduction from renaming,
    set consensus, and so on…

Thank you!