Memory Efficient Exploration in Anonymous Networks

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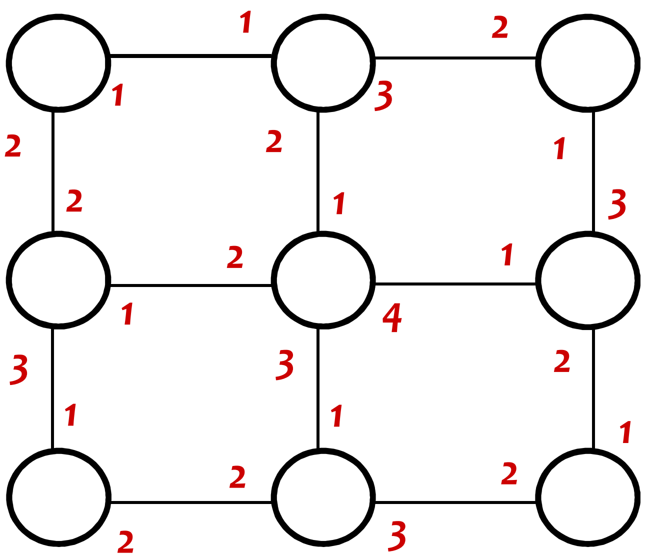
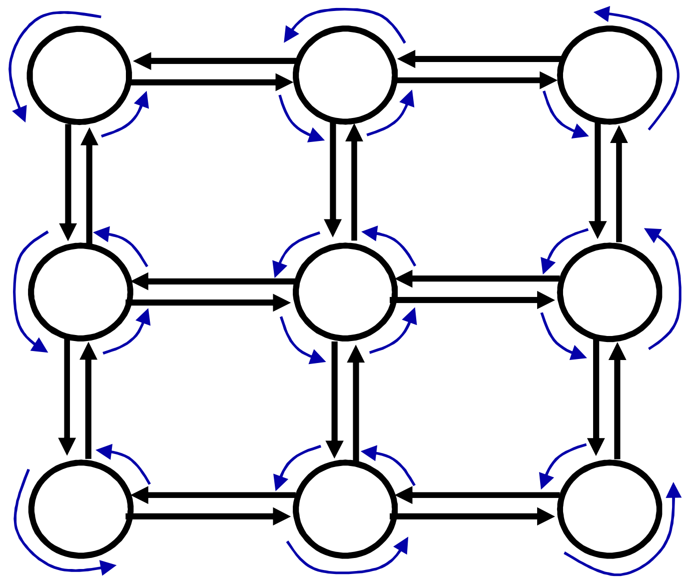
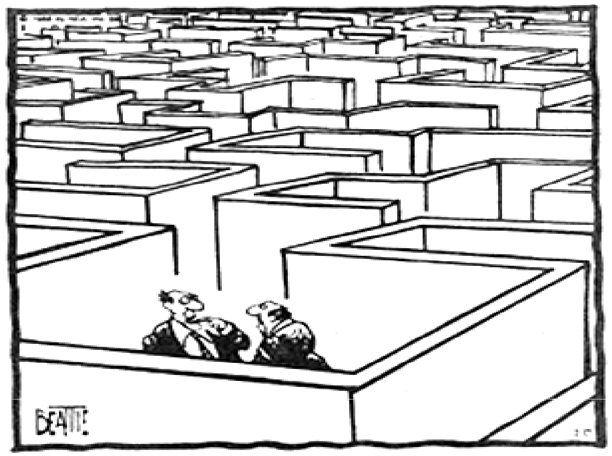
Anonymous Networks – labelled vs. implicit ports

- Equivalent definitions of anonymous graphs
  - with ***explicit*** and
  - ***implicit*** port ordering





Network/graph traversal problem



"The exit? Sure...take a right, then left, left again...no, wait...a *right*, then...no, wait...."

As *efficiently* as possible, typical complexity measures:

- ***memory utilization***,
- ***exploration time***,
- *use of* ***other resources*** *(markers, pebbles, colors, etc)*.

The random walk procedure

- The ***random walk***, is a mathematical formalization of a trajectory that consists of taking ***successive steps*** in *random directions*.
- A fundamental model for a ***random process*** in time. E.g., the following processes can be modeled as random walk
  - path traced by a molecule in a liquid or a gas (***Brownian motion***),
  - search path of a foraging animal,
  - price of a fluctuating stock and
  - financial status of a gambler, …
- A random walk on a graph is also a special case of a ***Markov chain***

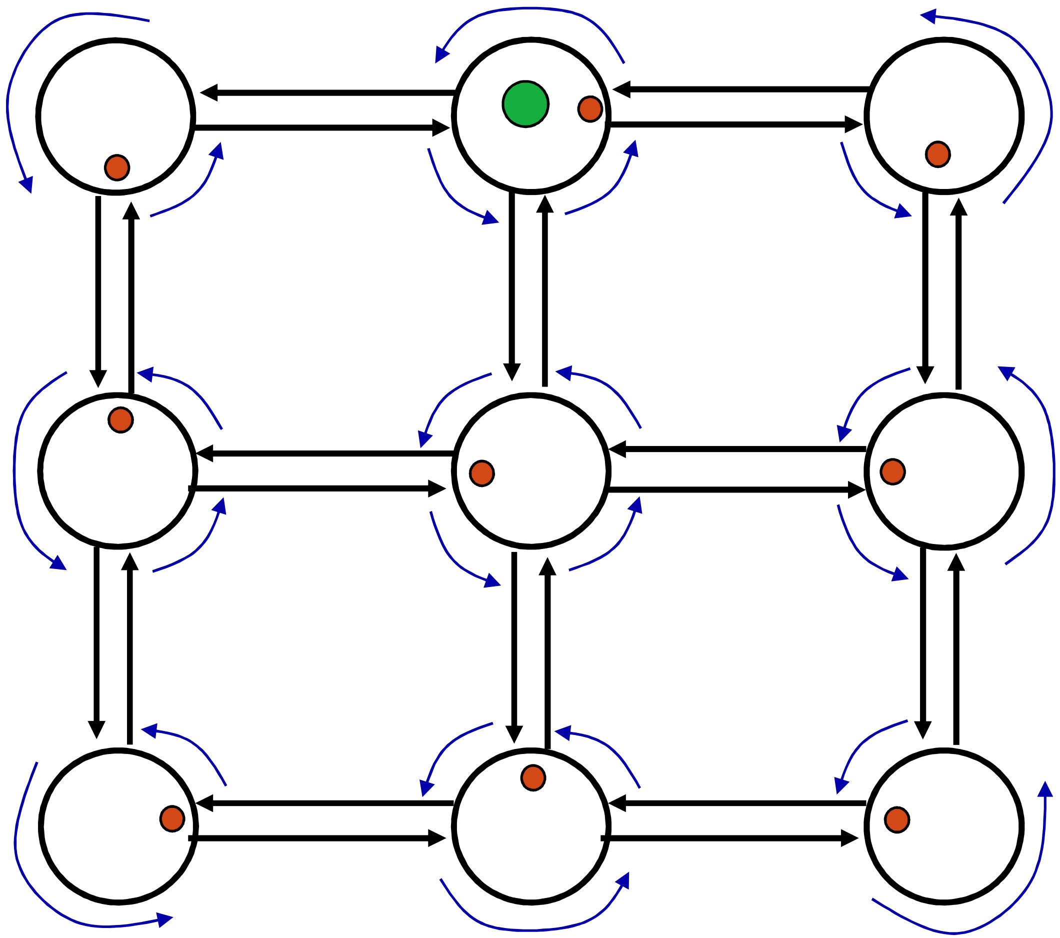
Basic results on the random walk

- Robot performing a random walk in an arbitrary graph of size ***n*** visits ***all nodes*** in the graph in (expected) time $O(n^3)$
    - R. Aleliunas, R.M. Karp, R.J. Lipton, L. Lovasz, and C. Rackoff, *FOCS'79*
- Robot performing a random walk in expected time:
    - complete graphs $O(n \log n)$
    - lines, trees $O(n^2)$
    - torus, 2D-grids $O(n \log^2 n)$
        - *(this can be improved to $O(n \log n)$ if **n** is known)*
- Robot performing a random walk in an arbitrary graph of size ***n*** visits ***all nodes*** in the graph in (expected) time $O(n^2 \log n)$ if we give preference to neighbours with lower degrees
    - S. Ikeda, I. Kubo, N. Okumoto, and M. Yamashita, *ICALP'03*

Deterministic counterparts for RW

- Traversal based on the random walk is virtually ***memory-less***, however it requires a ***large volume of (pseudo) random bits***
- There has been already a substantial attempt to study ***deterministic alternatives*** to the random walk
- Several models have been proposed and studied including:
  - ***the rotor-router mechanism*** and
  - ***the basic walk procedure***
- However, only a few results are known and further studies in the field would be highly appreciated

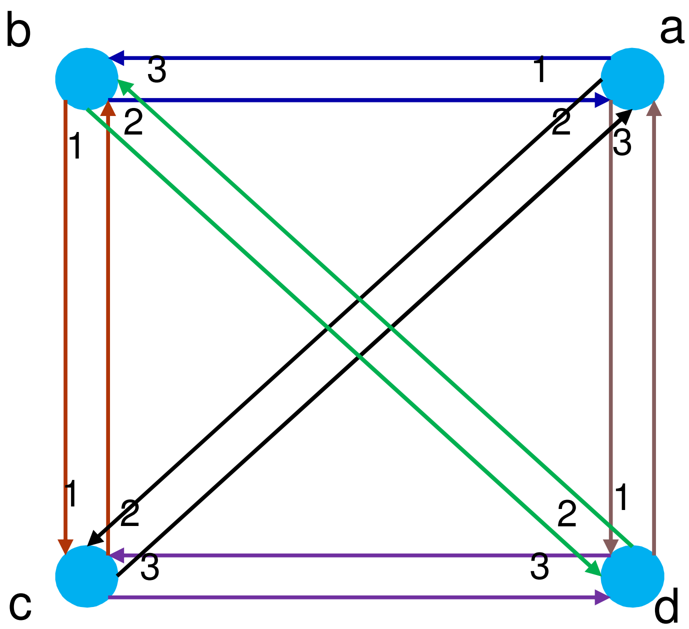
Rotor-router mechanism

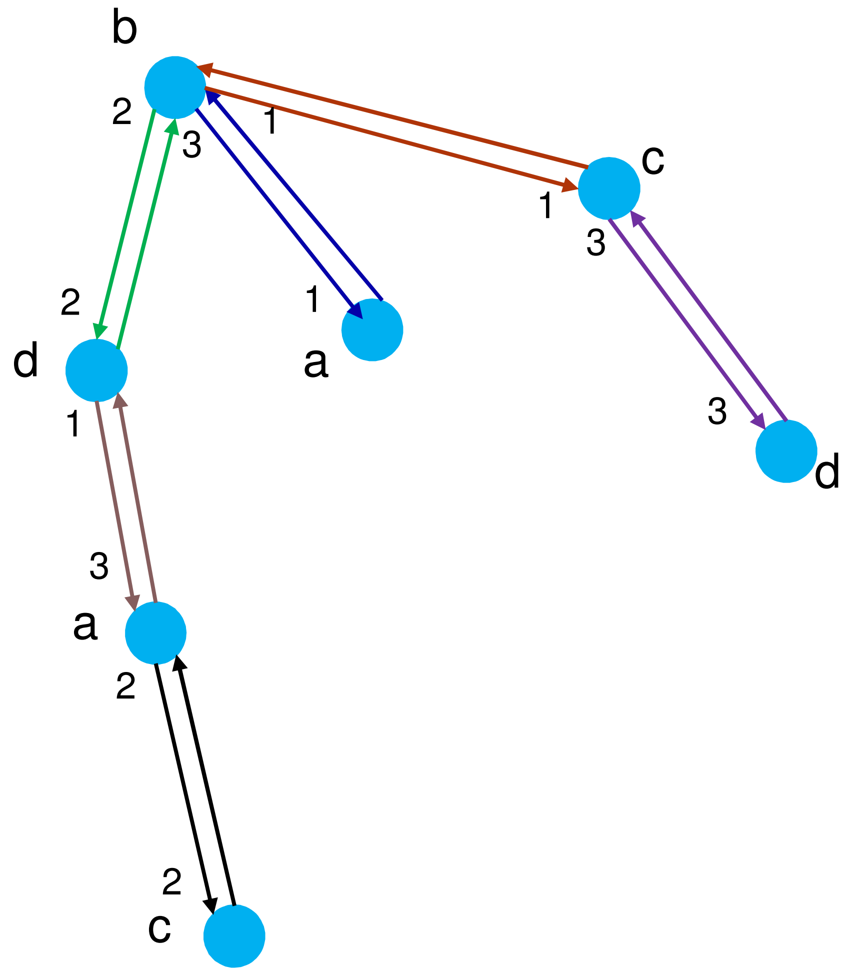


Traversal in rotor-router mechanism

- ***Robot locks*** in an Eulerian cycle in $O(V \cdot E)$ steps
  - S. Bhatt, S. Even, D. Greenberg, and R. Tayar,
    J. of Graph Algorithms and Applications'02
- ***Robot locks*** in an Eulerian cycle in $2 \cdot E \cdot D$ steps
  - V. Yanovski, I.A. Wagner, and A.M. Bruckstein,
    Algorithmica'03
- There is more and more work on comparison of performance of random walk and rotor-routers, e.g., in load balancing mechanism, infinite graphs, etc

Rotor-router model – Euler cycle





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abc$\boxed{\text{bdacadbabcdc}}$bdacadba...

Traversal in rotor-router mechanism

- Dependence of the lock-in time on the initial configuration of the rotor-router mechanism
  - Bampas, Gąsieniec, Hanusse, Ilcinkas, Klasing, and Kosowski, ***DISC'09***
- Min and max values of the lock-in time in considered cases

Scenario	Worst case	Best case
P-all	$\Theta(m)$	$\Theta(m)$
A(↻)P(℗)	$\Theta(m)$	$\Theta(m)$
P(℗)A(↻)	$\Theta(m \cdot \min\{\log m, D\})$	$\Theta(m)$
A(℗)P(↻)	$\Theta(m \cdot D)$	$\Theta(m)$
P(↻)A(℗)	$\Theta(m \cdot D)$	$\Theta(m)$ *for all* $D \leq n^{1/2}$
A-all	$\Theta(m \cdot D)$	$\Theta(m \cdot D)$

Traversal in rotor-router mechanism

- We show that after establishing an ***Eulerian cycle***

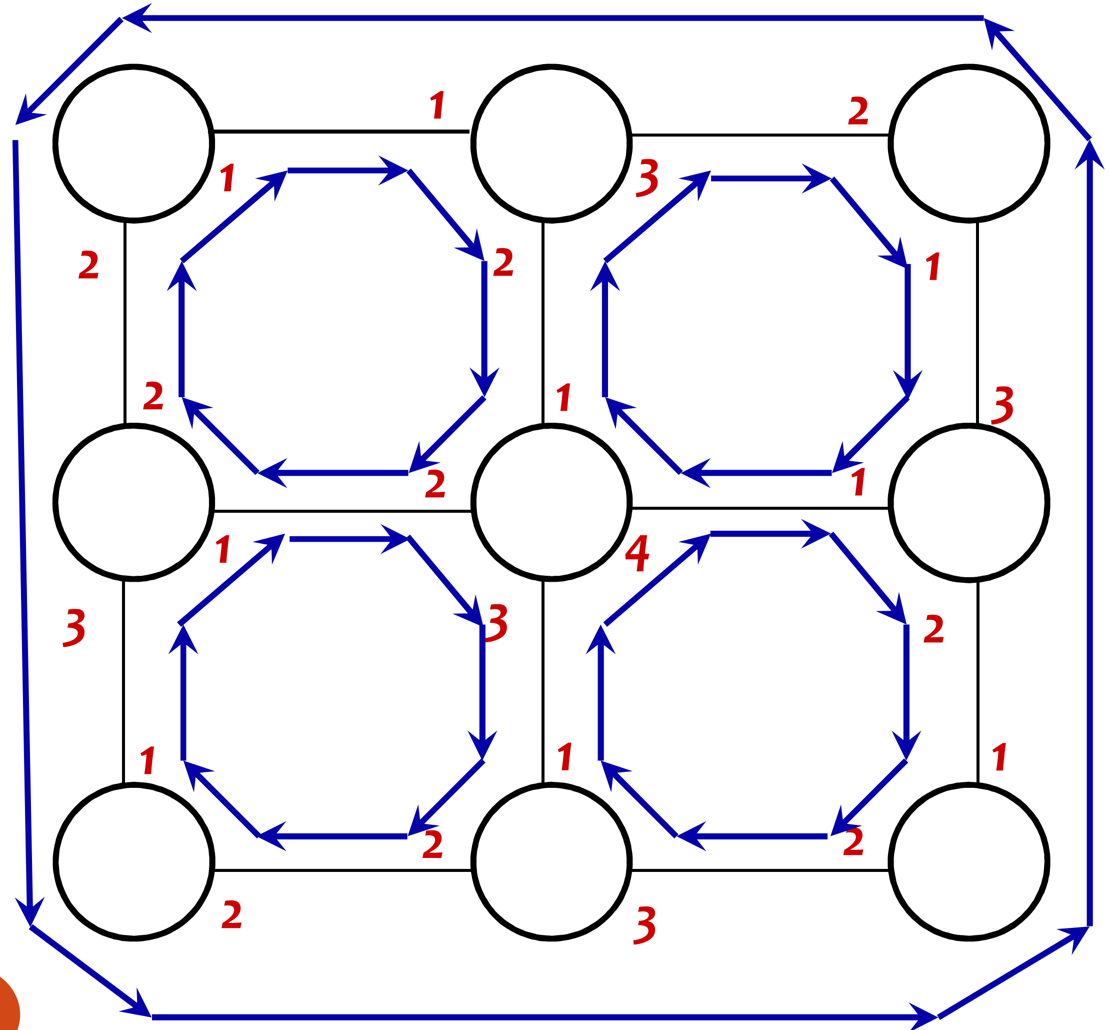
  Bampas, Gąsieniec, Klasing, Kosowski, and Radzik, ***OPODIS'09.***

  - (i) if at some step the values of ***k*** pointers v are arbitrarily changed, then a ***new Eulerian cycle*** is obtained within $O(k \cdot m)$ steps;
  - (ii) if at some step ***k*** edges are added to the graph, then a ***new Eulerian cycle*** is established within $O(k \cdot m)$ steps;
  - (iii) if at some step an edge is deleted from the graph, then a ***new Eulerian cycle*** is established within $O(\gamma \cdot m)$ steps, where γ is the number of edges in a shortest cycle in graph G containing the deleted edge.
- The results are based on the relationship between Eulerian cycles and spanning trees known as the "BEST" Theorem (due to de Bruijn, van Aardenne-Ehrenfest, Smith and Tutte)

Basic walk

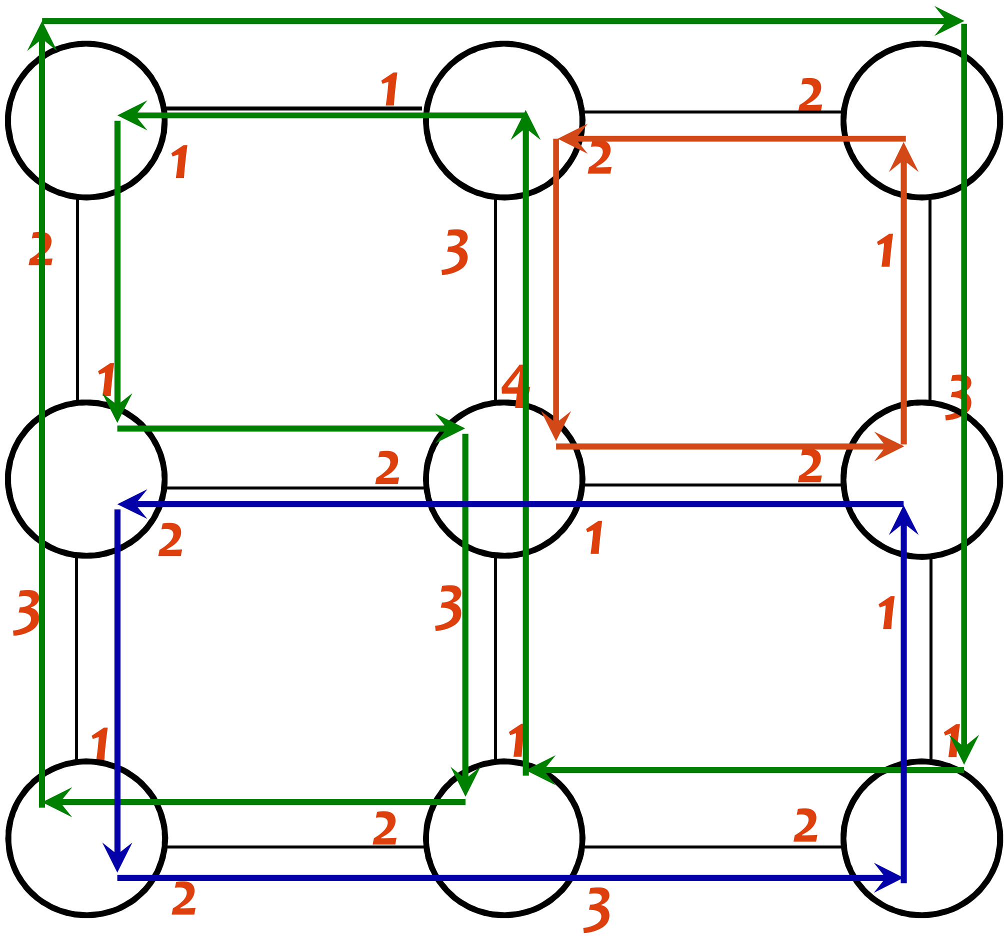
- This type of an algorithm can be used in case when the robot is barely equipped in the internal memory, i.e., the use of ***none*** or a ***constant number*** of memory bits is allowed.
- Simple actions of the robot are ***pre-programmed*** and could be seen as actions of a finite state machine, also ***the ports in the graph are pre-processed***.
- The task is to design a route based on port numbers and navigation abilities of the finite state machine that allows the robot to ***visit all graph nodes*** periodically.

Basic walk – cover by directed cycles



- The basic walk idea and an ***arbitrary arrangements of port numbers*** partitions all unidirectional edges (obtained from replacing each undirected edge by a pair of arcs with the opposite directions) into a ***number of directed cycles***

Basic walk - a tour



- In this model periodic graph exploration refers to ***arrangement of ports***, s.t., at least ***one tour containing all nodes*** in the graph is formed.

Comment: what about random ordering of ports? *It seems that the expected length of a cycle is ≈ 79.*

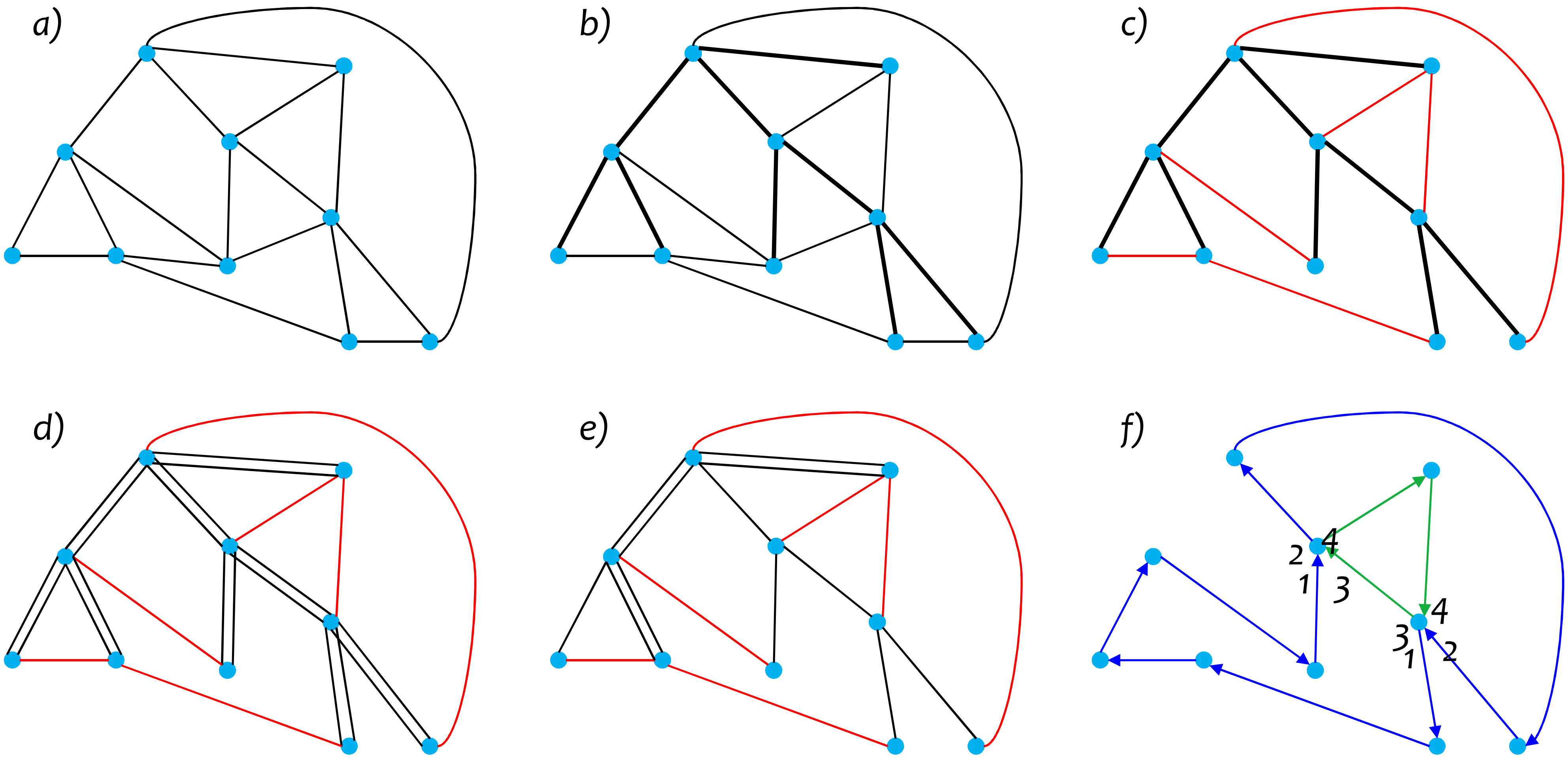
Oblivious robots, tour < 2n

In graphs having a spanning tree with non-saturated nodes

An input graph G

Find a spanning tree

Pick single edges



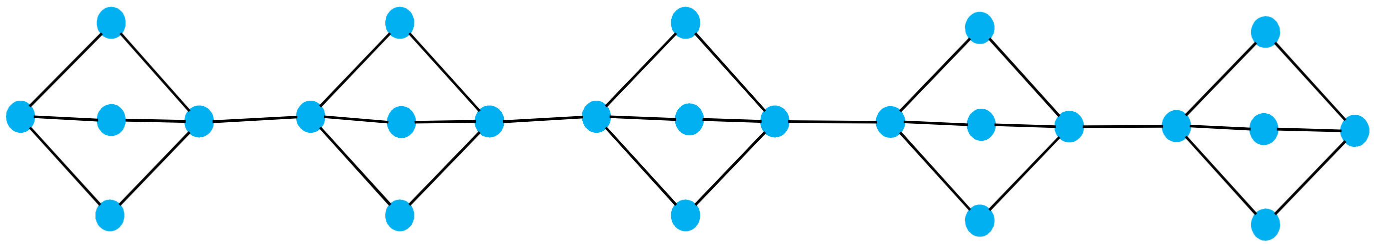
Double tree edges

Restore parity at nodes and remove double edges

One cycle of length < 2n

Oblivious robots, summary

- ***Searching for spanning trees*** with external graph edges at each node of the tree is ***NP-hard***. This problem is equivalent to finding a ***Hamiltonian cycle*** in ***cubic graphs*** (known to be NP-hard).
- ***Not every graph*** have a spanning tree with the ***desired property***, thus in general a different approach is needed.
- The best currently known ***bounds*** on the length of the periodic route used by oblivious robots are:
  - Upper ***4n***
  - Lower ***2.8n***

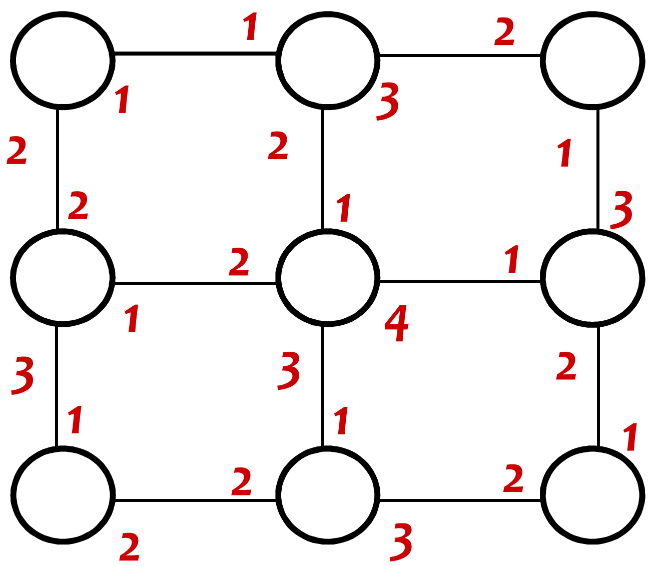


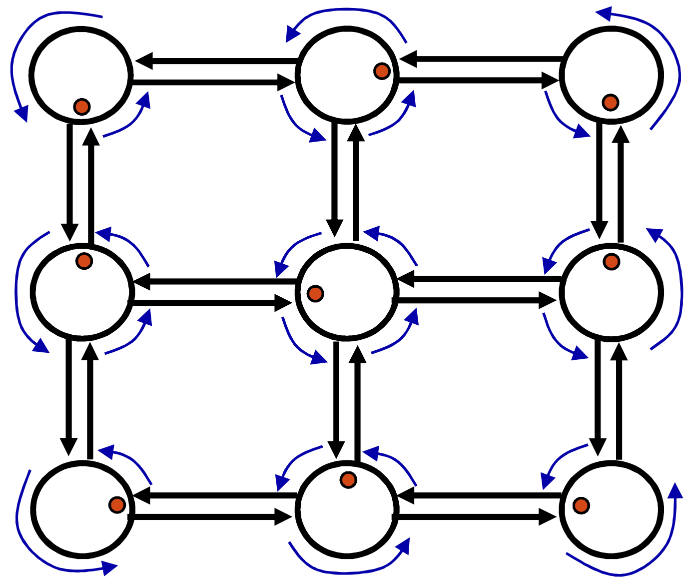
In this graph all edges must be traversed in two directions

Does extra memory help?

- In the model with implicit port numbers one needs to insert a ***fixed marker*** at one port of each node of the network.
- This breaks symmetry at the node and allows to use efficiently the memory provided to a robot

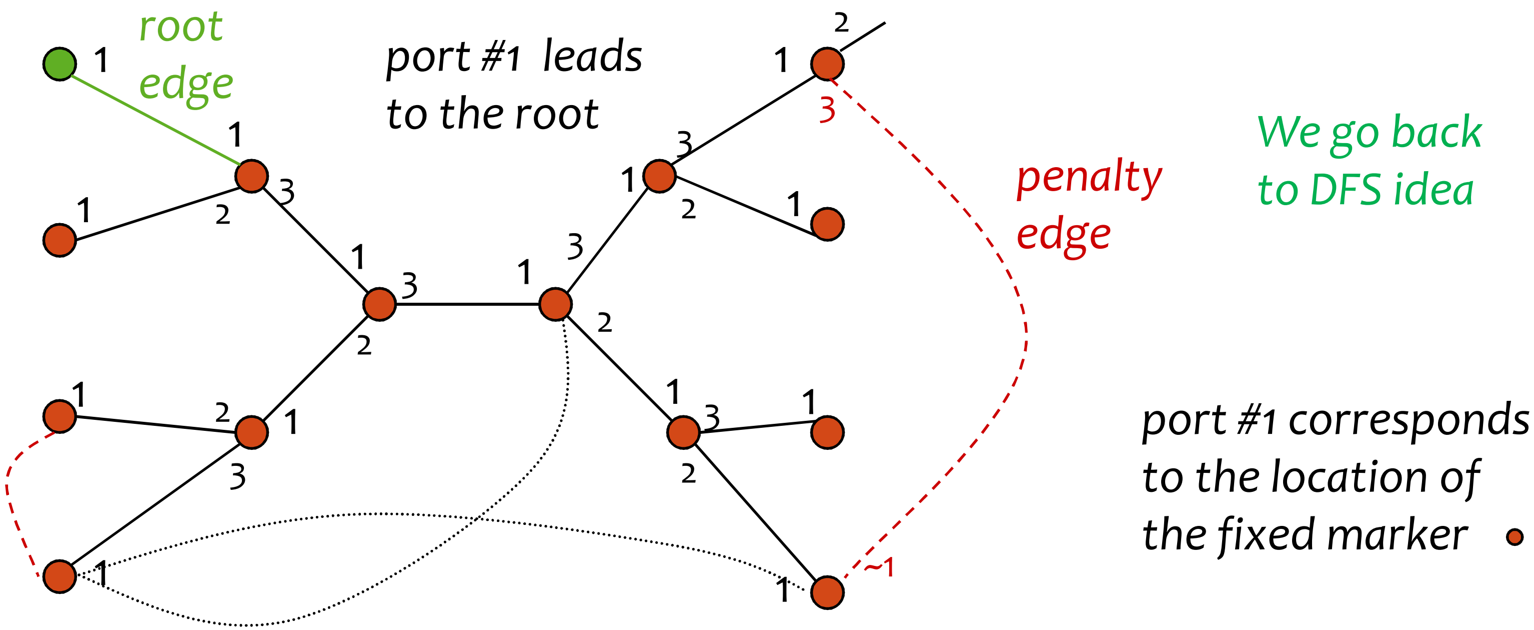
• *fixed marker*





Memory utilisation

- The exploration is performed along edges of a spanning tree encoded by port numbers.



Every node potentially carries ***a penalty edge***, thus the length of the tour is ≤ **4n-2**, where **2n-2** comes the spanning tree and **2n** from penalty edges. We know how to avoid at least n/4 penalty edges. This gives a tour of length at most ***3½n***.

Results in the basic walk model

- ***state-less*** graph exploration with the tour of length ***10n***
  - S. Dobrev, J. Jansson, K. Sadakane, W.-K. Sung, ***SIROCCO'05***
- ***2 bit-state*** exploration with the tour of length ***4n-2***; also *conjectured* lower bound of ***4n-O(1).***
  - D. Ilcinkas, ***SIROCCO'06***
- ***constant bit-state*** exploration with the tour of length ***3.75n-2.***
  - L. Gąsieniec, R. Klasing, R. Martin, A. Navarra, X. Zhang, ***SIROCCO'07***
- ***state-less*** exploration with the tour of length ***4.3(3)n*** and

  constant bit-state exploration with the tour of length ***3.50n-2.***
  - J. Czyzowicz, S. Dobrev, L. Gąsieniec, D. Ilcinkas, J. Jansson, R. Klasing, Y. Lignos, R. Martin, K. Sadakane, W.-K. Sung, ***SIROCCO'09***
- ***state-less*** exploration with the tour of length ***4n*** and
  - A. Kosowski and A. Navarra, ***MFCS'09***.

Other related problems

- Rendezvous problems
- Asynchronous computation/communication

Summary and further work

- *Model with preprocessed port numbers*
  - *(basic walk) oblivious robots* **2.8n … 4n**
  - *(basic walk) robots with constant memory* **2n … 3.5n**
- *(Model with the worst case port numbers*
  - *(rotor router) exact bounds on stabilization in various graph classes*
  - *(random walk vs. rotor router) exploration similarities/differences*
- *Model with random port numbers*
  - *(rotor router) performance in different classes of graphs*
  - *(random walk) performance in different classes of graphs*
  - *(basic walk) distribution of cycles, how many possible tours?*
  - *study of hybrid models, e.g., random rotor-router*
- *Multi-robot problems*
  - *Graph exploration, rendezvous and gathering, asynchronous agents, etc*