The Computational Power of Oblivious Mobile Robots

Forming a series of geometric patterns

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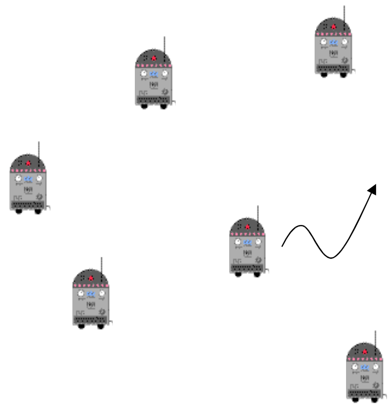
Team of Stupid Robots

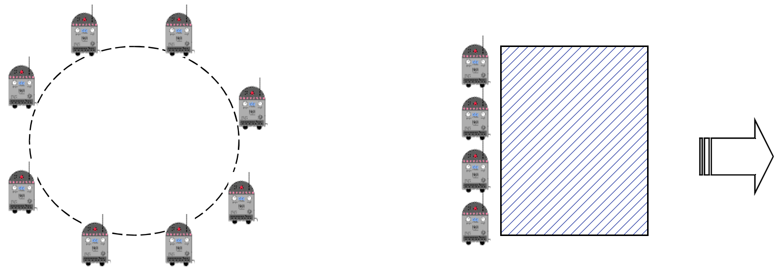
- Autonomous Robots
- Moving on a plane
- Can not communicate
- Have Vision (unlimited)
- Have no memory
- Semi-synchronous



Which tasks are possible for a team of stupid robots?

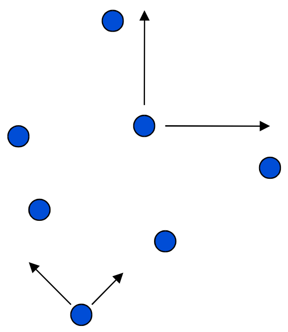
Pattern Formation

Can the robots arrange themselves on a ***circle*** ? on a ***line*** ?
or other geometric patterns?



Robot Model

- Points on the Plane [Dimensionless]
- Repeat
  - LOOK : Positions of robots
  - COMPUTE : new location
  - MOVE : to the computed location



- No agreement on Orientation
- No agreement on unit distance
- Identical algorithms
- Robots may be active or passive in ant time step

Models of Synchrony

- **Synchronous Model**

Robots act in synchronized time steps; Start at same time;

- **Semi-Synchronous Model [SYm]**

In each time step, a subset of robots are active;

Active robots complete exactly one LOOK-COMPUTE-MOVE step.

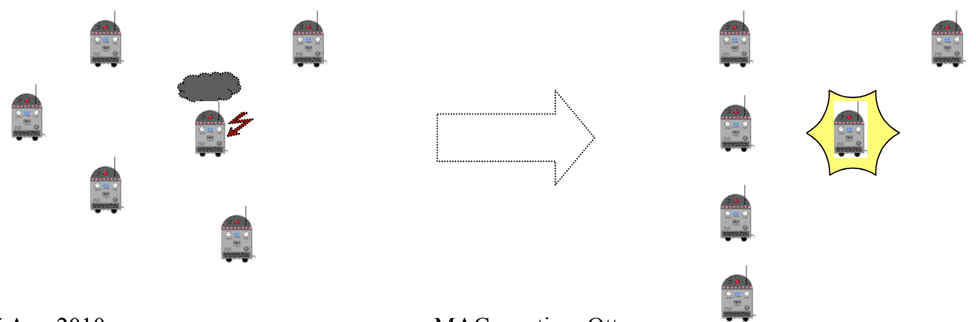
- **Asynchronous Model [CORDA]**

Robots complete each step in an arbitrary amount of time.

Motivation

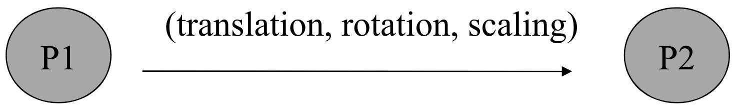
Why Oblivious ?

- Robots can **crash** (and recover at a later time).
- Robots may join at any time, in any state.
- Simple to design and analyze algorithms!



Arbitrary Pattern Formation (APF)

- **Pattern :** A set of points $\in$ IR^2
- Isomorphic Patterns : P1 $\sim$ P2

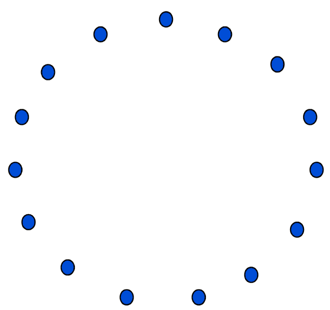


- A team of robots *forms a pattern* P if,
  There is some coordinate system Z, such that :
  Locations of robots = points in P (according to Z)

[*Multiple robots may occupy the same point.*]

Known Results

- **Circle Formation:** [Sugihara & Suzuki 1990]



a := farthest neighbor
b := closest neighbor
c := midpoint of the line segment (a,b)
If (distance to c) > |bc|
    move one step towards c
If (distance to c) < |bc|
    move one step away from c
Else, move one step away from b

Known Results

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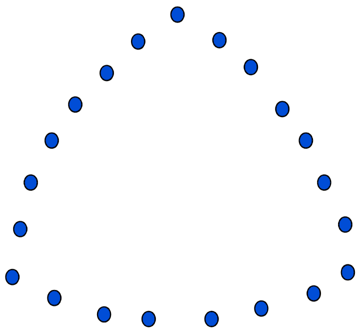

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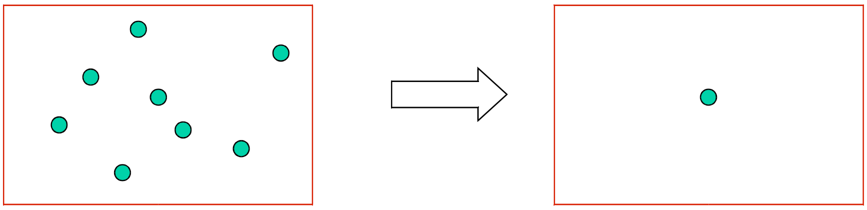
Circle Formation

- [Sugihara & Suzuki 1990] : Approximate Circle
- [Katreniak 2005] : Non-uniform Circle
- [Dieudonné & Petit 2007] : Uniform Circle ($k \neq 4$)
- [Défago & Souissi 2008] : Convergence to Uniform Circle
- [Suzuki & Yamashita 1999, 2010] : Uniform Circle

POINT formation (gathering)

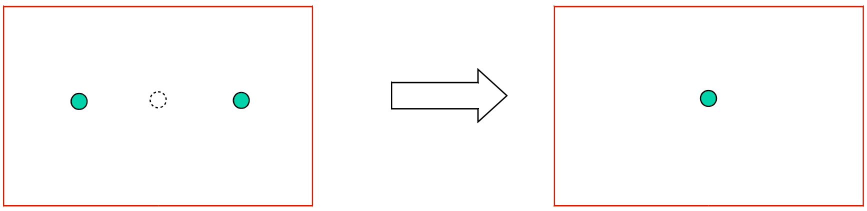


Theorem **[SY 1999]**
For **k = 2** oblivious robots, it is **impossible** to form POINT.

However, the two robots can converge to a point!

Theorem **[SY 1999]**
For **k >= 2** non-oblivious robots, POINT formation is always possible.

POINT formation (gathering)



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For **k = 2** oblivious robots, it is **impossible** to form POINT.

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Theorem **[SY 1999]**
For **k >= 2** non-oblivious robots, POINT formation is always possible.

Oblivious *vs.* Non-Oblivious

Arbitrary Pattern Formation

Theorem [SY 2010]

For k >=3 and any arbitrary pattern P,

k ***non-oblivious*** robots can form P <=> k ***oblivious*** robots can form P

Oblivious robots are almost as powerful as non-oblivious robots!

Series of Patterns

- Can oblivious robots form a ***series*** of Patterns

$P_1 , P_2 , P_3 , P_4 , \ldots$

How to remember if we do not have memory?

A robot system forms the series **$< P_1, P_2, P_3, \ldots, P_m >$**

if it goes transforms through the series of configurations

$C_0\ C_1\ C_{2,} \ldots .\ C_i \ldots\ C_{j\ ,} \ldots .\ C_{x\ ,} \ldots .\ C_{y\ ,} \ldots .$

$P_1, \quad P_2, \quad P_3, \quad P_m$

Which series are formable?

Theorem: No terminating algorithm can form a **finite** series of patterns **< P_1, P_2, P_3, …, P_m >** , where m>1

Want to form the infinite series: **(P_1, P_2, P_3, …, P_m)$^\infty$**

A series **< P_1, P_2, P_3, …, P_m >** is ***formable*** <=>
There exists an algorithm for forming the infinite series
(P_1, P_2, P_3, …, P_m)$^\infty$

Our Model (Adversary)

- **Semi-Synchronous:**
  At each time step, the *adversary* decides which robots are active;
  [Fairness: Each agent is activated infinitely often]
- **Lack of Orientation**
  Adversary decides the local coordinate system of each robot at each step;
- **Arbitrary Initial Configuration**
  Adversary decides the initial location of the robots.

Our Model (assumptions)

- **Agreement on Clockwise Direction**
  Robots agree on clockwise / counter-clockwise

- **Multiplicity Detection**
  Robots can **count** the number of robots at each location.

Three Scenarios

- **Visibly Distinct Robots**
  - Each robot is distinct (has visible ID)!
- **Indistinguishable Robots with ID's**
  - Robots look identical but are given invisible ID's!
- **Anonymous Robots**
  - All robots look identical and execute the same algorithm!

Visibly Distinct Robots

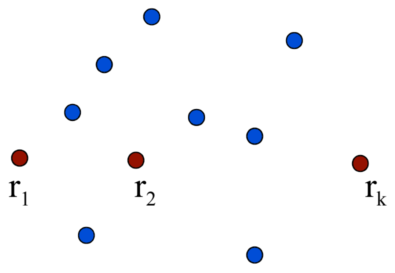
- k distinct robots

  $r_1, r_2, r_3, \ldots, r_k$

- To form pattern $\mathbf{P}_i$

Encode information as ratio of distances:

$$\frac{Dis(r_1, r_k)}{Dis(r_1, r_2)} = F(\mathbf{P}_i)$$



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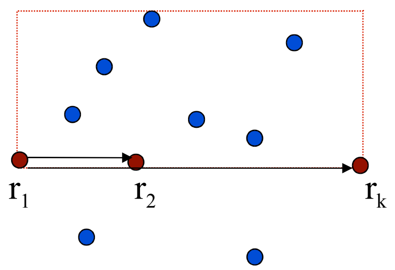
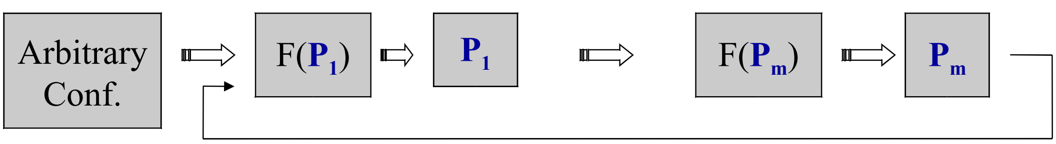
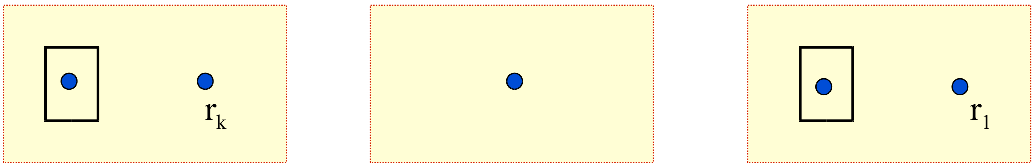


Robots can agree on a coordinate system!



Visibly Distinct Robots

- What about forming the **POINT** pattern?



Before POINT POINT After POINT

Visibly Distinct Robots

Theorem: The series **<P_1, P_2, P_3, …, P_m>** is formable iff

- size(P_i) <= k, for all i,
- P_i is non-isomorphic to P_j, $i \neq j$

Second Scenario

- **Visibly Distinct Robots**
  – Each robot is distinct (has visible ID)!
- **Indistinguishable Robots with ID's**
  – Robots look identical but are given invisible ID's!
- **Anonymous Robots**
  – All robots look identical and execute the same algorithm!

Indistinguishable Robots with ID's

- k distinct (but indistinguishable) robots

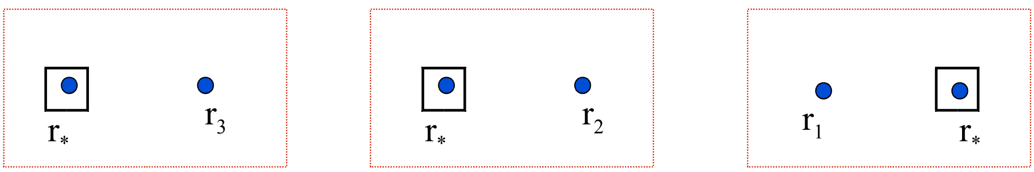
$r_1, r_2, r_3, \ldots, r_k$

Theorem: (impossibility)
With **k = 3** robots, the following series is not formable

$(P_1, P_2, \ldots P_l)$ [POINT] $(P_{l+1}, \ldots P_m)$ [TWO-POINTS]

where size(P_i) = 3, for all i. ($l < m$)

No robot can distinguish between



Indistinguishable Robots with ID's

Possibility:

Theorem: With **$k = 3$** robots, the series S^∞ is formable if S is a cyclic rotation of the following:

- (P_1, P_2, P_3, …, P_m) [POINT] [TWO-POINTS]
- (P_1, P_2, P_3, …, P_m)[TWO-POINTS][POINT]
- (P_1, P_2, P_3, …, P_m) [POINT]
- (P_1, P_2, P_3, …, P_m)[TWO-POINTS]
- (P_1, P_2, P_3, …, P_m)

where size(P_i) = 3, for all i.

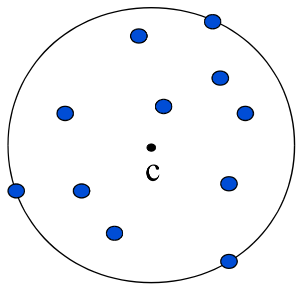
Indistinguishable Robots with ID's

Theorem: With **$k > 3$** robots,

$<P_1, P_2, P_3, \ldots, P_m>$ is formable iff

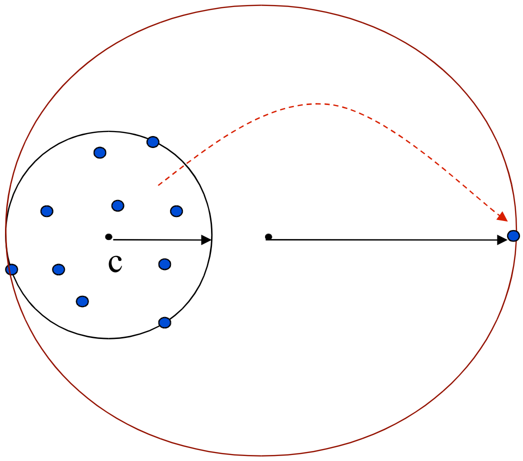
- size(P_i) <= k, for all i,
- P_i is non-isomorphic to P_j, i j

Algorithm for k > 3 robots



Smallest Enclosing Circle (SEC): [UNIQUE]

Algorithm for k > 3 robots



Bi-Circular Configuration (BCC):

Encodes info about a pattern using ratio of two radii

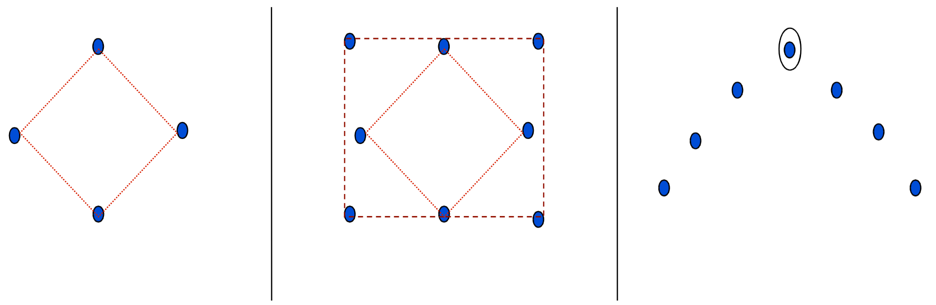
Third Scenario

- **Visibly Distinct Robots**
  - Each robot is distinct (has visible ID)!
- **Indistinguishable Robots with ID's**
  - Robots look identical but are given invisible ID's!
- **Anonymous Robots**
  - All robots look identical and execute the same algorithm!

Anonymous Robots

Assumption: The robots start from distinct locations.

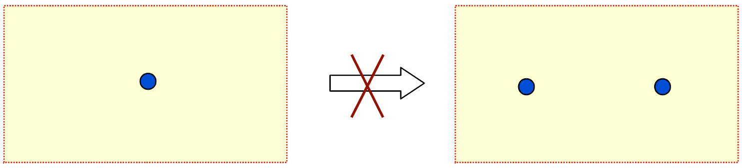
Robots in symmetric locations have similar view!



Symmetricity	$\rho = 4$	$\rho = 4$	$\rho = 1$
Size	$k = 4$	$k = 8$	$k = 7$

Anonymous Robots

Two robots in the same location may never be separated!



Necessary Condition:

In a series of patterns ($P_1, P_2, P_3, \ldots, P_m$)$^{\infty}$,
Sizes must be preserved, i.e.

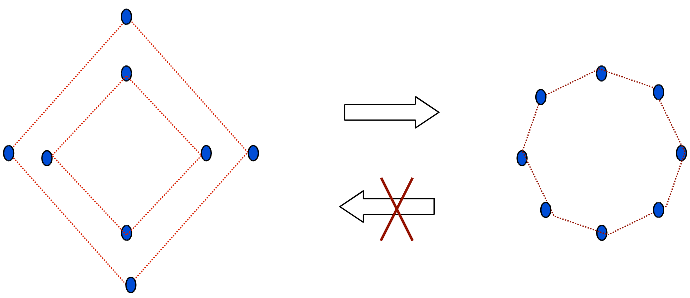
$$\text{size}(P_i) = \text{size}(P_j), \text{ for all } i,j$$

Anonymous Robots

Necessary Condition (2):

In a series of patterns ($P_1, P_2, P_3, \ldots, P_m$)$^{\infty}$,
The symmetricity must be preserved, i.e.

$$\rho(P_i) = \rho(P_j), \text{ for all } i,j$$



$\rho = 4$ $\rho = 8$

Anonymous Robots

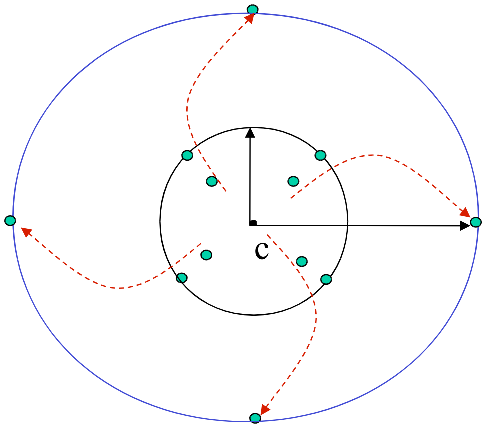
Sufficient Condition:

Theorem: The series **<P_1, P_2, P_3, …, P_m>** is formable, if

(1) $Size(P_i) = Size(P_j) = k$, for all i,j

(2) $\rho(P_a) = \rho(P_b) = q*\rho(C_0)$, for all a,b, (q >= 1)

Algorithm for Anonymous Robots



Q-Symmetric Circular Configuration SCC(q):

Encodes info about a pattern using ratio of two radii

Algorithm for Anonymous Robots

- The symmetry in a configuration is maintained only if the adversary activates all robots with same view, at the same time.
- In a symmetric configuration, form SCC(F($\mathbf{P_i}$)) to signal the formation of pattern P_i
- In an asymmetric configuration, form BCC(F(P_i)) to signal the formation of the next pattern P_i
- Ensure that every possible intermediate configuration contains information about the pattern to be formed.

To Summarize...

Characterization of formable series:

The series **$\langle P_1, P_2, P_3, \ldots, P_m \rangle$** is formable,

- With k visibly distinct robots if

  $\text{Size}(P_i) <= k$, for all i,

- With k indistinguishable ranked robots if

  $k > 3$ and $\text{Size}(P_i) <= k$

- With k anonymous robots if

  (1) $\text{Size}(P_i) = \text{Size}(P_i) = k$ and

  (2) $\rho(P_a) = \rho(P_b) = q*\rho(C_0)$, for all a,b, $(q >= 1)$

Conclusions

- We can overcome obliviousness of mobile robots not just for a single pattern but for a series of patterns.

Implement Global Memory without Local Memory!

- The technique of encoding information using ratio of distances can be used in other scenarios.
- These techniques can tolerate *measurement errors* up to certain extent.

Open Questions

- What happens when some robots are stalled permanently?
  - *Should the pattern include the failed robots?*
  - *How many failures can be tolerated?*
- What happens with robots have limited mobility?
  - *e.g. each robot moves unit distance in each step.*
- What happens in the case of limited visibility?

Thank you!