Self-deployment Algorithms for Mobile Sensors on a Ring

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Abstract. We consider the *self-deployment* problem in a ring for a network of identical sensors: starting from some initial random placement in the ring, the sensors in the network must move, in a purely decentralized and distributed fashion, so to reach in finite time a state of static equilibrium in which they evenly cover the ring. A self-deployment algorithm is *exact* if within finite time the sensors reach a static *uniform* configuration: the distance between any two consecutive sensors along the ring is the same, d; the self-deployment algorithm is ϵ-*approximate* if the distance between two consecutive sensors is between $d - \epsilon$ and $d + \epsilon$.

We prove that *exact* self-deployment is *impossible* if the sensors do not share a common orientation of the ring.

We then consider the problem in an oriented ring. We prove that if the sensors know the desired final distance d, then *exact* self-deployment is possible. Otherwise, we present another protocol based on a very simple strategy and prove that it is ϵ-approximate for any chosen $\epsilon > 0$.

Our results show that a *shared orientation* of the ring is an important computational and complexity factor for a network of mobile sensors operating in a ring.

1 Introduction

1.1 The Framework

A mobile sensors network is composed of a distributed collection of sensors that in addition to the traditional sensing, computation, and communication capabilities of static sensors, have also locomotion capabilities. Mobility facilitates a number of useful network capabilities; for example, they can patrol a wide area, they can be re-positioned for better surveillance, etc.; moreover, they are especially useful in environments that may be both hostile and dynamic. There have been some research efforts on the deploying of mobile sensors, most of them

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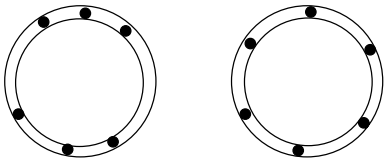

Fig. 1. Starting from an initial arbitrary placement (a), the sensors must move to a uniform cover of the ring (b)

based on centralized approaches; e.g., [22] assumes that a powerful cluster head is available to collect the sensor location and determine the target location of the mobile sensors.

Locomotion however allows the sensors to *self-deploy*; that is, starting from some initial random configuration, the sensors in the network can spread out in a purely decentralized and distributed fashion, and cover the area satisfying some optimization criteria (e.g., evenly, maximizing coverage, etc.) [11,12,13,15,21]. In contrast to [12] where the sensors are deployed one at the time, we consider the case when the sensors are deployed at the same time and they organize themselves in an adaptive manner. Unlike [15], we do not require prespecified destinations for the sensors, and unlike [12] we do not assume the sensors know where they are, since for small sensors localization is very hard. An essential requirement is that the network will reach a state of static equilibrium within finite time.

The *self-deployment* problem is quite similar to the *scattering* or *coverage* problem considered in cooperative mobile sensorics (e.g., [1]), and related to the *formation* problem (e.g. [10,18,19]); a key difference in these investigations is that usually there is no requirement that the network reaches a state of static equilibrium.

1.2 The Problem

In this paper, we are interested in the self-deployment of a mobile sensor network in a *ring* (e.g., a circular rim, as shown in Figure 1): starting from an initial random placement on the ring, the sensors must within finite time position themselves along the ring at (approximately) equal distance. A self-deployment algorithm, the same for all sensors, will specify which sequence of operations (communication/sensing, computing a destination, moving towards a point) a sensor must perform whenever it is active. We say that a self-deployment algorithm is *exact* if within finite time the sensors reach a *uniform* configuration: the distance between any two consecutive sensors along the ring is the same, d. We say that a self-deployment algorithm is ϵ-*approximate* if the distance between two consecutive sensors is between $d - \epsilon$ and $d + \epsilon$.

A self-deployment algorithm has recently been developed for the *line* [5] (e.g., a rectilinear corridor), and one has been designed for the *ring* as part of a larger protocol for uniform circle formation [2,6,14,17,20]. Both protocols yield only *approximate* solutions. However, they operate even with very weak sensors: anonymous (i.e., the sensors are indistinguishable), oblivious (i.e., each sensor has no memory of past actions and computations), asynchronous (i.e., each sensor becomes active at unpredictable times and the duration of its actions is unpredictable), and without a common coordinate system (e.g., no access to GPS). To date, no *exact* solution exists for these types of sensors.

1.3 Our Results

We first prove a strong negative result. In fact, we prove that *exact* self-deployment is actually *impossible* if the sensors do not share a *common orientation* of the ring; notice that this is much less a requirement than having global coordinates or sharing a common coordinate system. This impossibility result holds even if the sensors have unlimited memory and unbounded computational power, and even if all their actions, when active, are instantaneous and their visibility/communication radius is unlimited.

Faced with this strong negative result, the interesting question becomes under what restriction the self-deployment problem can be solved with an exact algorithm. Since the impossibility result holds in absence of common orientation of the ring, we consider the problem in *oriented* rings.

We prove that, in an oriented ring, if the sensors know the desired final distance d, then *exact* self-deployment is possible. In fact we present a simple protocol and prove that it allows the sensors to deploy themselves uniformly along the ring in finite time. This positive result holds even for the weakest sensors: anonymous, oblivious, asynchronous, with no common coordinate system; it works correctly even when every sensor can "locate" only its two neighbors or when the sensors have only a fixed sensing radius $v > d$.

Finally we turn to the case of an oriented ring when the desired final distance d is unknown. We present another protocol based on a very simple strategy and prove that it is ϵ-approximate for any fixed $\epsilon > 0$. As in [3,4,5], the difficulty is not in the protocol but in the proof of its correctness. Also in this case, the protocol works even for the weakest sensors: anonymous, oblivious, asynchronous, with no common coordinate system. The algorithm works correctly even when every sensor can "locate" only its two neighbors or when the sensors have only a fixed sensing radius $v \geq 2d$.

In the last protocol, the strategy we use is *go-to-half*. Interestingly was shown by Dijkstra [7] that in the *unoriented* ring *go-to-half* does *not* converge, and hence can not be used for self-deployment[1]. In other words, as already shown by our impossibility result, our result stresses that a *shared ring orientation* is an important computational and complexity factor for a network of mobile sensors

[1] It does however converge in a *line* as recently shown by Peleg [5] with a very involved proof.

operating in a ring. For space constraints, some of the proofs will be omitted and can be found in [8].

1.4 Related Work

The self-deployment problem has been investigated with the goal to cover the area so to satisfy some optimization criteria (e.g., evenly, maximizing coverage, etc.) [11,12,13,15,21]. For example, in [21] the problem is to maximize the sensor coverage of the target area minimizing the time needed to cover the area. Typically, distributed self-deployment protocols first discover the existence of coverage holes (the area not covered by any sensor) in the target area based on the sensing service required by the application. After discovering a coverage hole, the protocols calculate the target positions of these sensors, that is the positions where they should move. Loo et al. [15] considered a system consisting of a number of cooperating mobile nodes that move toward a set of prioritized destinations under sensing and communication constraints; unlike them, we do not require prespecified destinations for the sensors. Howard et al. [12] address the problem of incremental deployment, where sensors are deployed one-at-a-time into an unknown environment, and each sensor uses information gathered by previously deployed sensors to determine its deployment location. They assume every sensor is equipped with an ideal localization sensor. We do not assume the sensors know where they are, since for small sensors localization is very hard. The goal is to maximize network coverage under the constraint that nodes maintain line-of-sight with each other.

The self-deployment problem is related to a well studied problem in the field of autonomous mobile sensors: that of the *pattern formation* [9,10,18,19]; in particular to the one of *uniform circle formation* [2,6,17]. In this problem, very simple sensors are required to uniformly place themselves on the circumference of a circle not determined in advance (i.e., the sensors do not know the location of the circle to form). The main difference between these robotics investigations and our self-deployment problem in the ring is that in those problems, the sensors can freely move on a two dimensional plane; in contrast, our sensors can move only on the ring.

The strategy *go-to-half*, that we employ in one of our protocols was first analyzed by Dijkstra [7]; he showed that in an *unoriented* ring *go-to-half* does *not* converge (and hence can not be used for self-deployment). Recently, *go-to-half* has been shown by Peleg [5] (with a very involved proof) to converge in a *line*. Convergence in the unoriented ring has been announced for the *go-to-half-half* strategy by Défago and Konagaya [6,17].

2 Terminology and Model

We consider a sensors network in a ring (i.e., a circular line). Let $s_1, \ldots, s_n$ be the n sensors initially randomly placed on the ring (Figure 1). Let $d_i(t)$ be the distance between sensor s_i and sensor s_{i+1} at time t. When no ambiguity arises, we will omit the time and simply indicate the distance as d_i.

We will use a very general definition of a sensor as a computational unit capable of sensing (e.g., by communication) the positions of other sensors in its surrounding (within a fixed radius), performing local computations on the located/communicated data, and moving towards the computed destination. The local computation is done according to a deterministic algorithm that takes in input the located/communicated data, and returns a destination point towards which the sensor moves. All the sensors execute the same algorithm.

Each sensor repeatedly cycles through four *states*: when active, a sensor determines the positions of the other sensors in its radius – *Locate*; it computes the next destination point by executing the algorithm – *Compute*; and it moves towards the computed point – *Move*; after such a move the sensor may become inactive – *Wait*. The sequence: *Wait* - *Locate* - *Compute* - *Move* form a *computation cycle* (or briefly *cycle*) of a sensor. In the following, the "view of the world" of a sensor is defined as a snapshot of the positions of the other sensors in its own coordinate system (obtained in the *Locate* state).

The sensors are completely *autonomous*: no central control is needed. Furthermore they are *anonymous*, meaning that they are a priori indistinguishable by their appearance, and they do not (need to) have any kind of identifiers that can be used during the computation. They are *oblivious*: each sensor has no memory of past actions and computations; in other words, the computation is based solely on what located in the current cycle.

In general, no assumptions on the cycle time of each sensor and on the time each sensor takes to execute each state of a given cycle are made. It is only assumed that each cycle is completed in finite time, and that the distance traveled in a cycle is finite. Moreover, the sensors do not need to have a common notion of time, and each sensor can execute its actions at unpredictable time instants: this scenario is called *asynchronous* (ASYNC).

We also consider (in our impossibility result) a different scenario, where there is a global clock tick reaching all sensors simultaneously, and a sensor's cycle is an instantaneous event that starts at a clock tick and ends by the next. This scenario is called *semi-synchronous* (SSYNC). The only unpredictability is given by the fact that at each clock tick, every sensor is either *active* or *inactive*, and only active sensors perform their cycle. The unpredictability is restricted by the fact that at least one sensor is active at every time instant, and every sensor becomes active at infinitely many unpredictable time instants.

Let us denote by $\mathbb{AS}$ and $\mathbb{SS}$ the class of problems that are solvable in the asynchronous and in the semi-synchronous setting, respectively. Then,

Theorem 1 ([16]). $\mathbb{AS} \subset \mathbb{SS}$.

3 Impossibility of Exact Self-deployment

In this section, we show that the exact self-deployment problem is unsolvable; in other words, given a set of sensors placed on the rim of a circle, there exists no deterministic algorithm that, in a finite number of cycles, places the sensors uniformly on the ring.

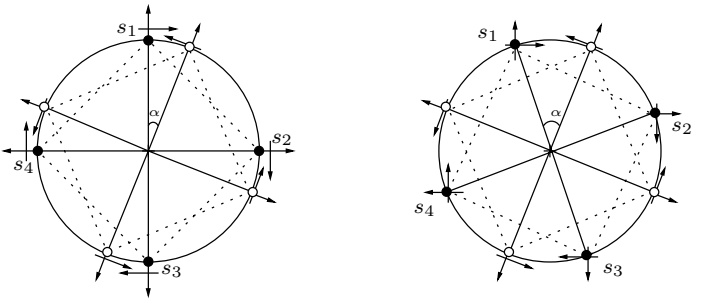

Fig. 2. (a) An example of starting configuration for the proof of Theorem 2. The black sensors are in S_1, while the white ones in S_2. (b) Theorem 2: the adversary moves only sensors in S_1.

Theorem 2. *Let $s_1, \ldots, s_n$ be all on a ring $\mathcal{C}$. Then, in* SSYNC*, there is no deterministic exact self-deployment algorithm.*

Proof. By contradiction, let us assume there exists a deterministic algorithm $\mathcal{A}$ that solves the problem in a finite number of cycles. Furthermore, let us assume that there is an even number of sensors placed on $\mathcal{C}$, and that the n sensors can be split in two subsets according to their views of the world. In particular, in the first subset, call it S_1, there are $s_1, \ldots, s_{n/2}$, and in the second subset, call it S_2 the other sensors. The sensors in S_1 and S_2 are placed on the vertices of two regular $n/2$-gons, and the two polygons are rotated of an angle smaller than $360°/n$. Furthermore, all sensors have their local coordinate axes rotated so that they all have the same view of the world (refer to Figure 2.a for an example).

Lemma 1. *If activating only the sensors in S_1 no exact self-deployment on $\mathcal{C}$ is reached, then also activating only the ones in S_2 no exact self-deployment on $\mathcal{C}$ is reached.*

Lemma 2. *If activating only the sensors in S_1 an exact self-deployment on $\mathcal{C}$ is reached, then also activating only the sensors in S_2 an exact self-deployment on $\mathcal{C}$ is reached. Moreover, activating both sets no exact self-deployment on $\mathcal{C}$ is reached.*

In the following, we define an adversary so that $\mathcal{A}$ never succeed in solving the problem. Algorithm 1 reports the protocol followed by the adversary.

First we note that, by the way the adversary is defined and since the sensors in S_1 (resp. S_2) have the same view, these sensors will always move together (when all activated). In the following, we will prove by induction the following property $\mathcal{P}rop$:

> for all $t \geq 0$, the sensors all have the same view of the world and are not in an exact self-deployment on $\mathcal{C}$.

Algorithm 1. The Adversary

(a) If activating only the sensors in S_1 no exact self-deployment on $\mathcal{C}$ is reached. Activates all sensors in S_1, while all sensors in S_2 are inactive, and goto (c). Otherwise,
  (b) If activating only the sensors in S_2, no exact self-deployment on $\mathcal{C}$ is reached. In this case, it activates all sensors in S_2, while all sensors in S_1 are inactive, and goto (c). Otherwise, all sensors are activated, and goto (c).

(c) If activating only the sensors in S_2 no exact self-deployment on $\mathcal{C}$ is reached. In this case, it activates all sensors in S_2, while all sensors in S_1 are inactive, and goto (a). Otherwise,
  (d) If activating only the sensors in S_1 no exact self-deployment on $\mathcal{C}$ is reached. In this case, it activates all sensors in S_1, while all sensors in S_2 are inactive, and goto (a). Otherwise, all sensors are activated, and goto (a).

By construction, $\mathcal{P}rop$ is clearly true at $t = 0$. Let us assume it is true at a given time $t > 0$. We distinguish the possible cases.

1. If the check performed in (a) is true, then clearly at time $t + 1$ there is no exact self-deployment on $\mathcal{C}$. Furthermore, all sensors will still have the same view of the world (see the example depicted in Figure 2.b).
2. If the check performed in (a) is true, then rule (b) is executed. Two subcases can occur.
   3.1. If the check of rule (b) is false, then at time $t + 1$ there is no exact self-deployment on $\mathcal{C}$, and all sensors have the same view of the world.
   3.2. Otherwise, all sensors are activated at time t, and by Lemma 2 no exact self-deployment on $\mathcal{C}$ is reached at time $t + 1$.
3. Rules (c) and (d) are handled symmetrically to previous rules (a) and (b).

Therefore, there is no time $t' \geq t$ so that the sensors are in a exact self-deployment on $\mathcal{C}$, having a contradiction.

By Theorem 1, we have

Corollary 1. *Let $s_1, \ldots, s_n$ be all on $\mathcal{C}$. Then, in* ASYNC *there is no deterministic algorithm that brings them uniformly distributed on $\mathcal{C}$ in a finite number of cycles.*

4 Self-deployment in an Oriented Ring: Interdistance Known

In this section we assume that the final distance d between two sensors is known to them. Moreover, the sensors have a fixed visibility radius of $2d$ and they can only locate up to such distance.

4.1 The Algorithm

The algorithm is very simple: sensors asynchronously and independently observe clockwise at distance $2d$, then they position themselves at distance d from the closest observed sensor (if any).

Protocol UNIFORM KNOWN (for sensor s_i)

- Locate clockwise at distance $2d$. Let d_i be the distance to next sensor. If none, $d_i = 2d$.
- If $d_i \leq d$ do not move.
- If $d_i > d$ move clockwise and place yourself at distance d from s_{i+1}.

4.2 Correctness

We say that a sensor is *white* if its distance to the clockwise neighbor is greater than or equal to d. We say that a sensor is *gray* if such a distance is smaller than d. Moreover we say that a white sensor is *good* if its distance to the clockwise neighbor is exactly d, it is *large* if its distance is strictly greater than d.

We call a *white bubble* a sequence of consecutive white sensors delimited by grey sensors. Let $W = s_i, s_{i+1}, \ldots, s_{i+m}$ be a white bubble. Sensor s_{i-1} is said to be the predecessor of the bubble, sensor s_{i+m+1} is the successor. Clearly predecessors and successors of a white bubble are gray, unless the ring contains white sensors only; notice that in this case all sensors are good. The size of W , indicated as $|W|$ is the number of white sensors composing the bubble (in this example m), its length, indicated by $l(W)$, is the length of the ring between the predecessor of the white bubble and its successor (assuming not all sensors are white); i.e., $l(W) = \sum_{j=-1}^{m} d_{i+j}$. Similarly, we define a *gray bubble* $G = s_i, s_{i+1}, \ldots, s_{i+m}$ as a sequence of consecutive gray sensors delimited by white sensors. Its size $|G|$ is the number of gray sensors in G; the length $l(G)$ is defined as the length of the ring between the first and the last gray sensor in G (note that this definition is different from $l(W)$).

The next two lemmas contain some simple facts.

Lemma 3. *At each point in time, if there are gray sensors, then the number of white bubbles equals the number of gray bubbles.*

Lemma 4. *At each point in time, if there are grey sensors there must be at least a bubble (i.e., a large sensor).*

Lemma 5. *A white sensor cannot become gray.*

Proof. In order for a white sensor s_j to become gray, its distance to the next sensor s_{j+1} should become smaller than d. By definition, sensors move clockwise and move according to the algorithm; so sensor s_{j+1} will never get closer to s_j. On the other hand, by definition of our algorithm, sensor s_j will never move at a distance smaller than d to s_{j+1}.

Lemma 6. *Let $W = s_i, s_{i+1}, \ldots s_{i+m}$ be a white bubble in the ring at time t. If $l(W) \geq d \cdot (|W|+1)$, in finite time, say at time t', the size of the bubble increases.*

Lemma 7. *Let $W_1, \ldots W_z$ be the white bubbles present in the ring at time t. At least one of these bubble W_k is such that $l(W_k) \geq d \cdot |W_k| + 1$.*

By Lemmas 6 and 7, we have that:

Lemma 8. *The number of grey sensors decreases.*

Finally, by Lemmas 5 and 8 we derive the main theorem.

Theorem 3. *In finite time all sensors are* good.

5 Self-deployment in an Oriented Ring: Interdistance Unknown

In this section we assume that each sensor has a fixed visibility radius of v, and does not know the final interdistance d between the sensors. Although d is not known, we must have that $v > 2d$ for our algorithm to work.

5.1 The Algorithm

Also this algorithm is very simple: sensors asynchronously and independently locate in both directions at distance v, then they position themselves in the middle between the closest observed sensor (if any).

Protocol UNIFORM UNKNOWN (for sensor s_i)

- Locate around at distance v. Let d_i be the distance to next sensor, d_{i-1} the distance to the previous (if no sensor is visible clockwise, $d_i = v$, analogously for counterclockwise).
- If $d_i \leq d_{i-1}$ do not move.
- If $d_i > d_{i-1}$ move to $\frac{d_i + d_{i-1}}{2} - d_{i-1}$ clockwise.

5.2 Correctness

Let $d_{min}(t) = Min\{d_i(t)\}$ and $d_{max}(t) = Max\{d_i(t)\}$. Let C be the length of the circumference of the ring. First observe the following simple fact:

Lemma 9. *We have that: $\forall t$, $d_{min}(t) \leq d$ and $d_{max}(t) \geq d$.*

Proof. By contradiction. Let the minimum distance be greater than d. We would have that $C > k \cdot d$, which is impossible since by definition $C = k \cdot d$. Same argument holds for d_{max}.

The next lemma shows that if, at some point there is a unique minimum (resp. maximum) interval, it will become bigger (resp. smaller).

Lemma 10. *If at time t there is a unique minimum interval, we have that: $\forall t, \exists t' > t : d_{min}(t') > d_{min}(t)$. If at time t there is a unique maximum interval, we have that: $\forall t, \exists t' > t : d_{max}(t') < d_{max}(t)$.*

Proof. Let s_{j-1} and s_j be the sensors that delimit the minimum interval $[s_{j-1}, s_j]$, whose length is $d_{j-1}(t) = d_{min}(t)$ at time t. First observe that, since $d_{j-2}(t) > d_{j-1}(t)$, by the algorithm we know that sensor s_{j-1} does not move at time t; actually, it will not be able to move as long as d_{j-2} remains greater than d_{j-1} (i.e., as long as s_j does not move). Consider now the first time t' when s_j is activated. Since s_{j-1} has not moved from time t to time t', we have that, at time t', $d_{j-2}(t')$ is still greater than $d_{j-1}(t')$. At time t', s_i then moves following the rule of the algorithm and $d_{j-1}(t') = \frac{d_{j-1}(t)+d_j(t')}{2} \geq \frac{d_{j-1}(t)+d_j(t)}{2} > d_{j-1}(t)$. Similar argument holds for d_{max}.

We now show that if at some point there are several minimum (resp. maximum) intervals of a certain length, their number will decrease.

Lemma 11. *If at time t there are $r > 1$ minimum intervals of length $d_{min}(t)$, either all intervals have length d and the sensors are deployed, or there exists a time $t' > t$ when the number of minimum intervals of length $d_{min}(t)$ is $r' < r$.*

Analogously,

Lemma 12. *If at time t there are $r > 1$ maximum intervals, either all intervals have length d and the sensors are deployed, or there exists a time t' when the number of maximum intervals is $r' < r$.*

We now show that the minimum intervals converge to a value $A = d - \gamma_{min}$, with $\gamma_{min} \geq 0$, and the maximum intervals converge to a value $B = d + \gamma_{min}$, with $\gamma_{max} \geq 0$.

Lemma 13. *Let $d_{min}(t)$ (resp $d_{max}(t)$) be the distance of a minimum (resp. maximum) interval at time t. We have that, for any arbitrary small $\epsilon > 0$ there exists a time $t' > t$ such that, $\forall t'' > t'$: $|d_{min}(t'') - A| \leq \epsilon$, and, $\forall t'' > t'$: $|d_{max}(t'') - B| \leq \epsilon$.*

Proof. From Lemmas 10 and 11 the intervals must converge; from Lemma 9 the minimum must converge to a value smaller than (or equal to) d, and the maximum must converge to a value greater than (or equal to) d.

Let us call *A-regular* at time t an interval that, at time t is ϵ-close to A; that is an interval whose length $d_j(t)$ is such that $|d_j(t) - A| \leq \epsilon$. Analogously, we call *$B$-regular* an interval that is ϵ-close to B. We call *A-irregular* at time t an interval that, at time t, is smaller than d, but not ϵ-close to A; *B-irregular* one that is greater than d, but not ϵ-close to B.

The following lemma shows that there exists a time t, after the time when the previous Lemma 13 holds, when any interval greater than the minimum (and smaller than d) is *A-regular*, and any interval smaller than the maximum (and

greater than d) is *B-regular*. In other words, each interval is either ϵ-close to A or to B. Notice that this property is not obvious; in fact, the only thing we know up to now is the convergence to A and B of the minimum/maximum intervals over time, while nothing is known about the other intervals.

Lemma 14. *Let $\epsilon > 0$ be arbitrarily small, and let t'_ϵ be a time when Lemma 13 holds. There exists a time $t''_\epsilon > t'_\epsilon$ when: for all intervals $[s_j, s_{j+1}]$ with $d_j(t'') \leq d$, $|d_j(t''_\epsilon) - A| \leq \epsilon$; for all intervals $[s_i, s_{i+1}]$ with $d_i(t''_\epsilon) \geq d$, $|d_i(t'') - B| \leq \epsilon$.*

Lemma 15. *Let t be a time when Lemma 14 holds. If at some time $t' > t$ at least an interval becomes irregular, then there exists a time $t'' > t'$ when all intervals are irregular.*

We now show that, after a time when Lemma 14 holds, all intervals actually converge to d (i.e., $A = B = d$).

Lemma 16. *Let $\epsilon > 0$ be arbitrarily small, and let t'_ϵ be a time when Lemma 14 holds. If $B - A > 2\epsilon$, at least an interval becomes irregular.*

Theorem 4. *For any arbitrary small $\epsilon > 0$ there exists a time t, such that $\forall t' > t, \forall i$: $|d_i(t') - d| \leq \epsilon$.*

Proof. By contradiction. Let $A \neq B$. From Lemma 14, there is a time t when all intervals are ϵ-close to A and B. From Lemma 16, at least one interval will become irregular at some time $t' > t$. However, by Lemma 15 there is a time $t'' > t'$ when all intervals become irregular (including the minimum and the maximum). This contradicts Lemma 13.

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